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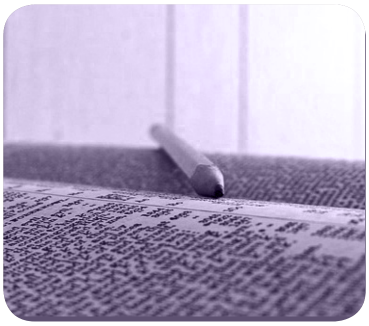
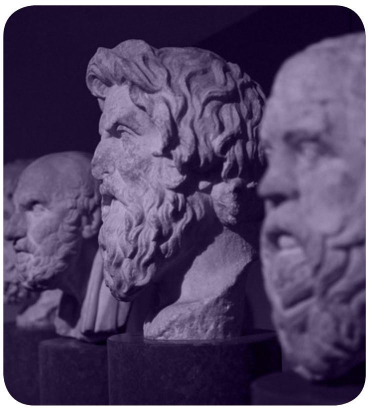
Inflationary potentials from the DBI action

Óscar Pozo Ocaña



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Inflationary potentials from the DBI action

Memoria de Trabajo de Fin de Máster realizada por

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Abstract

In this thesis we study an inflationary model in the context of string theory, in which the inflaton is identified with the position of a D7-brane in the internal space. The inflaton potential is obtained from the DBI and CS actions, which govern the dynamics of the D7-brane in the supergravity limit, after performing a dimensional reduction to the usual 4d space-time. Suitable background fluxes are considered to achieve single-field inflation, following known results in the literature. In these references a warped geometry of the 10d manifold is sourced by the fluxes, but a constant warp factor $Z = 1$ is considered. We analyze the general case in which the warp factor has a periodical dependence on the coordinates of the internal space and use the experimental bounds from CMB observations to impose constraints on its parameters.

Contents

1 Introduction

In the last decades there has been a great effort to obtain experimental data that characterize the universe in which we live. Experiments like the Wilkinson Microwave Anisotropy Probe (WMAP) [1] and more recently the *Planck* satellite [2] have measured with high precision the anisotropies of the Cosmic Microwave Background (CMB), obtaining tiny perturbations of order $\delta T/T \sim 10^{-5}$. Others like the Sloan Digital Sky Survey (SDSS) [3] have depicted a galaxy distribution that seems to be homogeneous on scales larger than a few megaparsecs. Finally, measurements of type Ia supernovae [4] indicate clearly that our universe undergoes an accelerated expansion.

The ΛCDM model [5] explains most of the current cosmological observations, simply considering a Friedmann-Robertson-Walker (FRW) metric and Einstein's equations to obtain all the dynamics. However, there are many shortcomings that this model does not address. On the one hand, different regions in the sky have the same temperature spectrum while they are expected to be out of causal contact at the time of CMB decoupling. This is known as the horizon problem. On the other hand, the combination of CMB and large-scale structure (LSS) data impose a flat spatial geometry for our universe, but the time evolution in this model drives it away from flatness. This is the so-called flatness problem. Both problems could only be solved considering highly-tuned initial conditions, so other mechanisms are needed in order to avoid this "accidental scenario".

The most famous and promising hypothesis is the existence of a period of exponential expansion in the early universe called cosmic inflation [6], [7], [8]. The approximately constant energy density of a scalar field, the inflaton, generates such a superluminal expansion. If inflation lasts long enough the horizon problem is potentially solved, since causal physics that established spatial homogeneity happen before inflation separates the different regions far away. Inflation turns out to drive the universe towards flatness and dilutes unwanted relics that are not observed presently. Moreover, quantum fluctuations of the inflaton can create small inhomogeneities that are stretched out during inflation, becoming eventually the seeds of the LSS and also of the small CMB anisotropies. The observation of such distributions is then used to check the validity of the inflationary models.

Although inflation is apparently the correct hypothesis, many questions remain about its physical origin. Nowadays there are two fundamental paradigms that explain most of the observed phenomenology of the universe. One of these frameworks is General Relativity [9], which deals with the force of gravity and interprets geometrically space and time as a unified object. The other is Quantum Field Theory, which describes the other three fundamental forces of nature that appear in particle physics. Despite their individual success, they are extremely different and there is not a theory of quantum gravity that combines them and explains the ultraviolet behavior of gravity. It turns out to be non-renormalizable, so at energies above the Planck scale, $M_P \simeq 2.4 \times 10^{18}$ GeV, divergences break unitarity and the theory makes no sense. In any model of inflation the existence of some non-renormalizable gravity interactions during the long-lasting expansion is guaranteed, so it is said that inflation is ultraviolet sensitive. Therefore, an ultraviolet completion is crucial to grasp all the dynamics of the inflationary process.

The usual approach is that new physics has to appear at some energy below the Planck scale. The strongest candidate is string theory, in which the fundamental objects are extended one-dimensional strings instead of point-like particles. The characteristic size l_s of the string, which is related to the string energy scale as $M_s = l_s^{-1}$, cuts off the gravity divergencies giving rise to a consistent framework of quantum gravity. Since it needs to live in a 10-dimensional manifold to avoid the conformal anomaly, six of those must form a compact space small enough so that it cannot be detected at low energies. The compactification that attains the usual 4-dimensional space-time brings many scalar fields coupled gravitationally, which are called moduli. They arise from the possible deformations of the shape and the size of the internal space, but they can also appear if other fundamental objects called Dp-branes are considered. They are $(p+1)$-dimensional hypersurfaces whose position, orientation and gauge-field configuration become moduli too. These dynamical objects are governed by the Dirac-Born-Infeld (DBI) and the Chern-Simons (CS) actions. In this thesis we study an inflationary model in which the inflaton is identified with the position of a D7-brane in the compact space, and its potential is extracted from the D7-brane dynamics.

There are many technical issues that make it extremely complicated to deal with string theory. However, in the context of inflation it is not necessary to consider the full theory. The typical scale of energies at which inflation happens is well below the usual string energy scale. This allows us to construct an inflationary model in the so-called supergravity limit, in which only the massless spectrum of states is considered due to the lack of energy needed to reach excited states. Moreover, the interactions of the inflaton with the other moduli are highly suppressed if a moduli stabilization mechanism provides them mass terms large enough. Constructing a model that presents this hierarchy of energies, inflation can be studied from the effective field theory point of view involving a reduced number of states. Moduli stabilization can be achieved introducing non-vanishing background fluxes for the field strengths of the massless string spectrum, and also with non-perturbative contributions to the action. These fluxes can bring different shapes for the inflationary potential, and also source a warped geometry for the 10-dimensional space-time. The purpose of this thesis is to generalize the computations of [10], [11] considering a non-constant warp factor with a periodical dependence on the inflaton field, analyze the effects that such oscillatory modes produce on the cosmological observables and put constraints on the form of these oscillations using cosmological data from CMB observations.

The present work is organized as follows. In section 2 we introduce the basic cosmology knowledge needed to study single-field inflation and to compare theoretical results with experimental data. In section 3 the string theory framework that will be used is presented, focusing on the flux compactification mechanism, the D-brane dynamics and the effective theory viewpoint. In section 4 the inflationary model is developed. First of all, the background of the model is established and the procedure to extract the inflaton potential from the DBI and CS actions is explained. The introduction of a non-constant warp factor is motivated afterwards, and the relation between the hierarchy of energies and the parameters of the model is discussed. In section 5 the numerical evaluation of the model is explained and its results are given for the different potential shapes considered with an analysis of the cosmological constraints over the non-constant warp factor. Finally, the conclusions of the thesis are presented in section 6.

2 Cosmology

2.1 Basics of General Relativity

As we have just discussed, there is experimental evidence that our universe is homogeneous and isotropic on large scales. Assuming such spatial symmetries, the most general form of the metric is given by the FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Cr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right) , \tag{2.1}$$

where ds^2 is the line element, $a(t)$ is a scale factor that can depend only on time, the variables r, θ, φ are the usual spherical coordinates and C is the curvature parameter for the spatial geometry[1]. We will assume for simplicity the flat case $C = 0$. Therefore, we find a background geometry that can evolve with time, changing the relative size of the universe. Its expansion rate can be studied through the Hubble parameter[2]

$$H \equiv \frac{\dot{a}}{a} . \tag{2.2}$$

Considering Einstein's General Relativity, the evolution of the scale factor $a(t)$ is driven by the energy-momentum tensor $T_{\mu\nu}$ of the universe through the famous equation[3]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu} , \tag{2.3}$$

where $g_{\mu\nu}$ is the four-dimensional space-time metric, $R_{\mu\nu}$ is the Ricci tensor and R the Ricci scalar. Then the matter content of the universe is what determines its evolution over time. The energy-momentum tensor that satisfies the large scale symmetries has the perfect fluid form

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & pg_{ij} \end{pmatrix} , \tag{2.4}$$

where ρ is the local energy density, p the local pressure and $i, j = 1, 2, 3$. Introducing (2.1) and (2.4) into (2.3), one obtains the following two independent equations for the evolution of the scale factor $a(t)$, which are called the Friedmann equations

$$H^2 = \frac{1}{3}\rho , \tag{2.5}$$

$$\dot{H} = -\frac{1}{2}(\rho + p) = \frac{\ddot{a}}{a} - H^2 . \tag{2.6}$$

A universe in which there is more than one form of matter/energy can also be considered. In that case, ρ and p are the local, total energy density and pressure, and they are just the sum of individual ones.

[1] $C = 0$ for a flat space and $C = \pm 1$ for a positively/negatively curved space.

[2] The overdot will be used to denote the time derivative, and $'$ will be used for the total derivative.

[3] We will take for simplicity $M_P = (8\pi G)^{-1/2} = 1$ and $c = 1$ in all the work.

2.2 Conditions for inflation

The main purpose of inflation is to avoid highly fine-tuned initial conditions. In order to solve issues like the horizon problem, inflation must give some explanation of how different regions of the universe which today are far away were in causal contact before, justifying the correlation of their physical properties. To analyze causality it is very useful to turn to the comoving particle horizon

$$\tau \equiv \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{a_1}^{a_2} d\ln a \left(\frac{1}{aH}\right) , \tag{2.7}$$

which is just the maximum distance[4] that light can travel between time t_1 and t_2. The factor $(aH)^{-1}$ is known as the comoving Hubble radius. The way it evolves with time is crucial to see if there is an increase or a decrease of the fraction of the universe in causal contact. If the comoving horizon decreases over time, it means that the different regions we observe today with a large separation could have been at short distances before. Then, one of the conditions that inflation must satisfy is given by

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 , \tag{2.8}$$

which implies directly an accelerated expansion of the universe

$$\ddot{a} > 0 . \tag{2.9}$$

This condition can be rewritten in terms of the slow-roll parameter ϵ, defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} , \tag{2.10}$$

and the condition simply states that $\epsilon < 1$. Equation (2.6) shows that we also need a negative pressure $p < -\rho/3$ to source this accelerated expansion. If ϵ is small enough then the Hubble parameter is nearly constant during inflation and equation (2.2) states that

$$a(t) \simeq a(t_0) e^{H\cdot(t-t_0)} , \tag{2.11}$$

which implies exponential expansion of the universe. However, these conditions are not enough to solve the horizon and flatness problems. The inflationary period must last enough to explain cosmological observations without requiring fine-tuning. Instead of a time interval, what is normally used is the number of e-folds, defined through the relation

$$N_e \equiv \ln\left(\frac{a(t)}{a(t_0)}\right) = \int_{t_0}^{t} dt'\ H(t') . \tag{2.12}$$

It is just another way to label the time evolution, which expresses the growth of the scale factor $a(t)$ in powers of e. The end of inflation is defined by

$$\epsilon_1(\phi_{end}) = 1 . \tag{2.13}$$

But determining the number of e-folds that inflation lasts is quite challenging since it depends on a huge part of the physics we observe presently. It is not only the flatness and the horizon problem. Post-inflationary physics like the period of reheating and the reproduction of the observed matter content of the universe constrain the possible number of e-folds, too. However, there is a general agreement that the inflationary period should last about $N_e = 50 - 60$ e-folds to solve all these problems [12], [13].

[4] Taking $c = 1$, it coincides with the definition of the conformal time.

2.3 Dynamics of single-field inflation

In the simplest models only one scalar field is considered to drive the inflationary period. However, when one tries to combine string theory with inflation there are many scalar fields that can do the job. Although inflation can be reached with multiple scalar fields dominating, we will consider the simple case of a single scalar field with a non-canonical kinetic term. The most general action describing a single inflaton ϕ is given by

$$S = -\int d^4x\sqrt{-g}\left\{\frac{R}{2} + \frac{1}{2}\chi(\phi)\partial_\mu\phi\partial^\mu\phi + V(\phi)\right\} \ , \tag{2.14}$$

where $\chi(\phi)$ is a function that depends on the inflaton and $V(\phi)$ is the potential term. Performing the field redefinition

$$\hat{\phi} = \int_0^\phi d\varphi\sqrt{\chi(\varphi)} \ , \tag{2.15}$$

an action with a canonical kinetic term is obtained for the scalar field $\hat{\phi}$. The dynamics of an inflaton with canonical kinetic term can be found in the basic inflationary literature [14]. However, there are some cases in which this integral cannot be evaluated analytically and $\hat{\phi}$ has to be found numerically. The kinetic term that we will find in this work is one of this type, so we discuss all the dynamics without using the canonical field $\hat{\phi}$ in order to obtain analytic expressions. It is sufficient to study the case in which the scalar field depends only on time, since the fast expansion smoothes out all the spatial variations rapidly. Applying the Euler-Lagrange formalism one obtains the following scalar field equation

$$\ddot{\phi} + \frac{1}{2}\chi^{-1}(\phi)\chi'(\phi)\dot{\phi}^2 + 3H\dot{\phi} + \chi^{-1}(\phi)V'(\phi) = 0 \ . \tag{2.16}$$

Varying the metric instead of the scalar fields, one obtains the Einstein equation (2.3) with local energy density and pressure given by

$$\rho = \frac{1}{2}\chi(\phi)\dot{\phi}^2 + V(\phi) \ , \qquad\qquad p = \frac{1}{2}\chi(\phi)\dot{\phi}^2 - V(\phi) \ . \tag{2.17}$$

Therefore, inflation requires this kind of unconventional matter content to dominate during the period of expansion. The Friedmann equations (2.5) and (2.6) become the following ones

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\chi(\phi)\dot{\phi}^2 + V(\phi)\right) \ , \tag{2.18}$$

$$\dot{H} = -\frac{1}{2}\chi(\phi)\dot{\phi}^2 \ . \tag{2.19}$$

This set of equations and (2.16) govern the inflationary background, and can be used to find the time evolution of the Hubble parameter H and the inflaton field ϕ. The dynamics obtained computing numerically the canonical field $\hat{\phi}$ are analogous, and they are a good resource to check the results obtained from the above equations.

2.4 The slow-roll limit

The slow-roll limit guarantees the inflationary period. The first condition states that the first slow-roll parameter ϵ defined in (2.10) is very small, so that the potential energy dominates over the kinetic energy

$$V(\phi) \gg \chi(\phi)\dot{\phi}^2 \ , \tag{2.20}$$

and then the Hubble parameter is nearly constant. The second condition involves the second slow-roll parameter

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \ , \tag{2.21}$$

and states again its smallness $|\eta| \ll 1$. Therefore, in this limit the inflaton field evolves very slowly along its potential, ensuring a long-lasting period of exponential expansion. Equations (2.16) and (2.18) simplify with these assumptions and become

$$0 \simeq 3H\dot{\phi} + \chi^{-1}(\phi)V'(\phi) \ , \tag{2.22}$$

$$H^2 \simeq \frac{1}{3}V(\phi) \ . \tag{2.23}$$

Therefore, it is a limit in which the Hubble slow-roll parameters can be computed using the kinetic and potential terms without the necessity of solving the time evolution of the system. For simplicity, we will give the formulas in terms of the canonically normalized field $\hat{\phi}$. Defining the variables

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{V'(\hat{\phi})}{V(\hat{\phi})}\right)^2 \ , \qquad \eta_V \equiv \frac{V''(\hat{\phi})}{V(\hat{\phi})} \ , \qquad \xi_V \equiv \frac{V'(\hat{\phi})V'''(\hat{\phi})}{V^2(\hat{\phi})} \ , \tag{2.24}$$

the slow-roll parameters are given by

$$\epsilon = \epsilon_V \ , \tag{2.25}$$

$$\eta = \eta_V - \epsilon_V \ , \tag{2.26}$$

$$\xi = \xi_V - 3\epsilon_V\eta_V + 3\epsilon_V^2 \ . \tag{2.27}$$

In the case in which this limit is satisfied, these relations are useful to perform simpler computations and also to check the results obtained with a numerical evaluation of the full equations of motion, which is a more intricate procedure.

2.5 Quantum fluctuations

The inflationary expansion of the universe dilutes any kind of previous inhomogeneity, but it is obvious that at some point such inhomogeneities must appear. Inflation can provide a physical origin for the initial seeds of all structure in the universe. The idea is to interpret the inflaton as a quantum field, so that it has quantum fluctuations that are stretched by the superluminal expansion. Since the inflaton is coupled to the metric, the quantum fluctuations of its energy density modify space-time, generating a spectrum of curvature perturbations. Space-time undergoes wave perturbations that can be decomposed in terms of different wavelengths. Once matter falls in the troughs of these waves, density perturbations start growing and collapsing gravitationally, generating the different large scale structures.

The inflationary period generates characteristic quantum modes for the curvature perturbations. To analyze them, a homogeneous background is considered with all the previous formalism and then matter and metric perturbations are introduced. Considering again a flat space-time, we expand

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(\mathbf{x},t) \ , \qquad g_{\mu\nu}(t,\mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\mathbf{x}) \ , \tag{2.28}$$

where

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + a^2(t)\left((1-2\Psi)\delta_{ij} + h_{ij}\right)dx^i dx^j \ . \end{aligned} \tag{2.29}$$

We focus on scalar and tensor fluctuations of the metric (Φ, Ψ, h_{ij}), which are the ones that we can observe presently as density fluctuations and gravitational waves. To avoid studying unphysical quantum perturbations, it is better to look at the ones that are gauge invariant. The relevant one that has been widely studied is the comoving curvature perturbation $\mathcal{R}$, which is given during inflation by

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi \ . \tag{2.30}$$

Metric perturbations generate a backreaction onto the field dynamics while inflaton perturbations backreact on the space-time metric. This leads to a complicated set of equations between the different perturbations [14], but they just come from generalizing the Einstein and Friedmann equations to this perturbed case. Looking at the scalar perturbations, the only equation that interests us is

$$v_k''(\tau) + \left(k^2 - \frac{z''(\tau)}{z(\tau)}\right)v_k(\tau) = 0 \ , \tag{2.31}$$

that arises after defining the Mukhanov variable

$$v \equiv z\mathcal{R} \ , \qquad \text{where} \qquad z^2 \equiv a^2(\tau)\frac{\dot{\phi}^2}{H^2} \ , \tag{2.32}$$

and after performing the Fourier expansion[5]

$$v(\tau,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} v_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} \ , \tag{2.33}$$

working with the conformal time instead of the usual time coordinate. Equation (2.31) can be solved numerically if the background has been determined previously, or can be solved analytically considering simplifications, for example assuming slow-roll conditions. However, it shows one of the main characteristics of the scalar perturbation $\mathcal{R}$. While the comoving wave number k is constant during inflation, the comoving Hubble radius $(aH)^{-1}$ decreases its value. When $k < aH$, it is said that the mode exits the causal horizon and equation (2.31) shows that for $k \ll aH$ the k^2 term can be ignored and we get a constant

[5]Note that equation (2.31) only depends on the modulus of $\mathbf{k}$ due to homogeneity and isotropy.

solution for the $v_k(\tau)$ mode. Therefore, the Fourier mode $\mathcal{R}_k = v_k/z$ remains constant outside the horizon and the k dependence can be interpreted as a scale dependence. For the tensor perturbations h_{ij} introduced in (2.29) we have the analogous situation. Differentiating their two polarizations h^+ and $h^\times$, we obtain the same equations for their Fourier modes replacing $z = a$.

2.6 Cosmological observables

Once the $\mathcal{R}_k$,h_k^+ and $h_k^\times$ modes are determined, the power spectra $\mathcal{P}_\mathcal{R}(k)$ and $\mathcal{P}_t(k)$ are the quantities that allow us to connect the theory with the experiment. They are correlators defined by

$$\mathcal{P}_\mathcal{R}(k) \equiv \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \ , \tag{2.34}$$

$$\mathcal{P}_t(k) \equiv \frac{k^3}{2\pi^2}\left(|h_k^+|^2 + |h_k^\times|^2\right) \ . \tag{2.35}$$

The large scale distribution of galaxies or the temperature anisotropies of the CMB depend crucially on the primordial power spectra produced at the end of inflation. This allows to put experimental bounds in some properties as the tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_\mathcal{R}(k)} \ , \tag{2.36}$$

and the spectral indices

$$n_s - 1 \equiv \frac{d\ln \mathcal{P}_\mathcal{R}(k)}{d\ln k} \ , \tag{2.37}$$

$$n_t \equiv \frac{d\ln \mathcal{P}_t(k)}{d\ln k} \ , \tag{2.38}$$

that parametrize the scale dependence of the power spectra. Inflation is studied observing temperature and density anisotropies on the sky but it is limited by the size of our horizon. After inflation our comoving horizon grows and all the fluctuations that were frozen eventually re-enter the horizon. The largest k modes have not entered yet, so they cannot be detected. In order to compare our theory with observations, the cosmological observables must be evaluated at horizon crossing[6], when $k = aH$. It is the point at 50-60 e-folds before the end of inflation, the point from which we can extract the information of the earliest observable universe. Perturbations at horizon crossing are imprinted in the CMB, and measurements have given recent bounds on the cosmological observables [15], [16] and also on the running of the scalar spectral index. To test our theoretical results we will use the following ones.

$$r \ < \ 0.07 \ , \tag{2.39}$$

$$n_s \ = \ 0.9667 \pm 0.0080 \ , \tag{2.40}$$

$$\frac{dn_s}{d\ln k} \ = \ -0.002 \pm 0.013 \ , \tag{2.41}$$

[6]The star subscript will be used to denote this point of horizon crossing.

all given with a 95% confidence level. These cosmological observables can be computed in terms of the Hubble slow-roll parameters

$$\epsilon_1 \equiv \epsilon \ , \qquad \epsilon_{i+1} \equiv \frac{\dot{\epsilon}_i}{H\epsilon_i} \ , \qquad i = 1, 2, \dots \ , \tag{2.42}$$

using the Green's function method [17]. The resulting expressions are the following ones.

$$r = 16 \times \left(\epsilon_1 + \epsilon_1^2 + 2(C+1)\epsilon_1\epsilon_2\right) \ , \tag{2.43}$$

$$n_s - 1 = -2\epsilon_1 - \epsilon_2 - 2\epsilon_1^2 - (2C+3)\epsilon_1\epsilon_2 - C\epsilon_2\epsilon_3 \ , \tag{2.44}$$

$$\frac{dn_s}{d\ln k} = -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 \ , \tag{2.45}$$

where $C \equiv \ln 2 + \gamma_E - 2 \simeq -0.7296$, γ_E is the Euler-Mascheroni constant. These cosmological observables can be also computed in the slow-roll limit in terms of the slow-roll parameters $\epsilon_V, \eta_V, \xi_V$ defined in (2.24). The resulting relations are

$$r = 16\epsilon_V \ , \tag{2.46}$$

$$n_s = 1 + 2\eta_V - 6\epsilon_V \ , \tag{2.47}$$

$$\frac{dn_s}{d\ln(k)} = 2\xi_V + 24\epsilon_V^2 - 16\eta_V\epsilon_V \ . \tag{2.48}$$

Finally, another important quantity that has been measured is the amplitude of scalar perturbations, A_s. The power spectrum is often expanded in terms of k as follows

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \ . \tag{2.49}$$

This quantity can be obtained in terms of the slow-roll parameters to first order through the relation

$$A_s = \frac{H^2}{8\pi^2\epsilon} \ , \tag{2.50}$$

also evaluated at horizon crossing. The experimental measurement given by *Planck* [15] is

$$A_s = (2.207 \pm 0.076) \times 10^{-9} \quad (68\%) \ . \tag{2.51}$$

3 String theory

3.1 The framework

As discussed in the introduction, string theory is an example of a theory that can give the ultraviolet completion needed for inflation. It is a generalization of Quantum Field Theory in which the fundamental objects are open and closed strings instead of point-like particles. The idea is that all particles we know emerge from the different vibrational modes of these extended, one dimensional objects. Their trajectory describes a worldsheet instead of the classical worldline. This worldsheet can be parametrized by two coordinates σ, τ, where $0 \leq \sigma \leq \pi$ is the spatial coordinate along the string and $\tau \in \mathbb{R}$ describes its time evolution. Therefore, the trajectory of the string is completely determined in the target space-time by the coordinates $x_M(\sigma, \tau)$, with $M = 0, 1, \ldots, d-1$ and d the dimension considered.

To avoid physical and mathematical inconsistencies the theory needs supersymmetry and it has to live in ten dimensions. With these conditions there are five possible string theories: Type I, Type IIA, Type IIB, and the heterotic $E_8 \times E_8$ and $SO(32)$ theories. In our case we consider the Type IIB supergravity theory, a chiral one in which there are only closed strings at the perturbative level. Non-perturbatively we have both closed and open strings as fundamental objects, and also the so-called D-branes. We will focus on the latter in section 3.4. The supergravity limit represents the effective field theory for energies below the mass scale M_s of the different modes of the superstring. Regarding the content of its spectrum, we focus on the massless bosons, which are divided in two sectors. In the NS-NS sector there are a scalar φ called the dilaton, an antisymmetric 2-form gauge field B_2, with field strength $H_3 = dB_2$, and the space-time metric G_{MN}. On the other hand, in the R-R sector we have three p-forms C_p with its corresponding field strength $F_{p+1} = dC_p$. They are C_0, C_2 and C_4. The bosonic part of the type IIB supergravity action is divided in three pieces

$$S_{IIB} = S_{NS} + S_R + S_{CS} \ , \tag{3.1}$$

which are given by

$$S_{NS} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} e^{-2\varphi} \left(R + 4\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}|H_3|^2 \right) \ , \tag{3.2}$$

$$S_R = -\frac{1}{4\kappa^2} \int d^{10}x \sqrt{-G} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) \ , \tag{3.3}$$

$$S_{CS} = -\frac{1}{\kappa^2} \int C_4 \wedge H_3 \wedge F_3 \ . \tag{3.4}$$

In these equations R is the curvature scalar, κ^2 is the 10-dimensional Newtonian coupling constant, related with the string tension[7] $T = (2\pi\alpha')^{-1}$ by $2\kappa^2 = (2\pi)^7(\alpha')^4$ and

$$\tilde{F}_3 = F_3 - C_0 H_3 \ , \tag{3.5}$$

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 - \frac{1}{2} B_2 \wedge F_3 \ . \tag{3.6}$$

[7]Here we define $\alpha' \equiv l_s^2$, so we have the relation $\alpha' = M_s^{-2}$.

Moreover, the self-duality condition of the 5-form $\tilde{F}_5$ has to be imposed

$$\tilde{F}_5 = *\tilde{F}_5 \ , \tag{3.7}$$

in order to avoid having twice the number of desired propagating degrees of freedom. It is usually convenient to work in the Einstein frame instead of the string frame, and we can go from one to the other via a Weyl rescaling $G_{MN} \to e^{-\varphi/2} G_{MN}$. Defining the forms

$$\tau \equiv C_0 + i e^{-\varphi} \ , \tag{3.8}$$

$$G_3 \equiv F_3 - \tau H_3 \ , \tag{3.9}$$

the supergravity action (3.1) can be rewritten in the Einstein frame as

$$S_{IIB} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G_E} \left(R_E - \frac{|\partial \tau|^2}{2\left(\mathrm{Im}[\tau]\right)^2} - \frac{|G_3|^2}{2\mathrm{Im}[\tau]} - \frac{|\tilde{F}_5|^2}{4} \right) - \frac{i}{8\kappa^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}[\tau]} \ . \tag{3.10}$$

Applying the Euler-Lagrange formalism one can obtain the equations of motion of this theory. There are other equations of motion for the forms C_p given by the Bianchi identities $d\left(*F_p\right) = 0$, where $*$ denotes the usual Hodge dual operation

$$*_d \, A_{\mu_1 \ldots \mu_{d-p}} \equiv \frac{1}{p!} \epsilon_{\mu_1 \ldots \mu_{d-p}}{}^{\nu_1 \ldots \nu_p} A_{\nu_1 \ldots \nu_p} \ . \tag{3.11}$$

3.2 Compactification, moduli and fluxes

An important question is how we deal with ten space-time dimensions if only four have been observed. The extra six dimensions must be hidden in some way to obtain consistent results. To solve this issue one assumes a ten-dimensional manifold with the product structure

$$\mathcal{M} = \mathcal{M}_{1,3} \times Y_6 \ , \tag{3.12}$$

where $\mathcal{M}_{1,3}$ is our usual $3+1$ space-time and Y_6 is a general six-dimensional hermitian manifold sufficiently small to have escaped detection so far. In order to preserve some degree of supersymmetry and chirality in 4d, Y_6 should be a so-called Calabi-Yau manifold. This kind of manifold is compact as a topological space. To be clear with the tensorial notation, the indexes $M, N = 0, 1, \ldots, 9$ will be used to denote the components of the 10-dimensional target space-time while $\mu, \nu = 0, 1, 2, 3$ will be used for the components in $\mathcal{M}_{1,3}$ and $u, v = 4, \ldots, 9$ for the ones in Y_6. Considering that the internal space is small compared to the typical length scales which we are going to work with in 4d, it can be integrated out so that we get an effective 4d theory. But these compactifications always come with a problem. There are continuous deformations of the Y_6 internal space that do not change the effective energy and therefore correspond to massless scalars in four dimensions, which are called moduli. Many of them appear in generic compactifications, and we will focus on the following ones

- Dilaton: It is the scalar field φ controlling the worldsheet perturbative expansion. In this work we will assume that it is constant, so it can be identified with the string coupling constant $g_s \equiv e^{\varphi}$.
- Axions: They appear when p-form potentials C_p are considered and Y_6 has non-trivial harmonic p-forms, so adding them to C_p does not modify the field strength F_{p+1}.
- Kähler and complex structure moduli: They are related to the size and shape of the internal space.

The existence of such massless scalars is also inconsistent with what we observe. They can couple at least gravitationally to matter so they could generate extra forces due to particle exchange, which is experimentally constrained. Therefore, we need mechanisms to create potentials for these moduli, giving them masses large enough to explain why we have not observed them yet. However, at high energy scales like during inflation, their presence is also problematic. They can spoil the exponential expansion due to the interactions with the inflaton. We have the freedom of choosing a vacuum for our theory and the challenge of finding one in which all moduli have large enough mass terms to ignore them, as explained in section 3.5. This procedure is known as moduli stabilization. Many of the moduli can be stabilized using fluxes. As a simple example, we consider the introduction of non-vanishing vacuum expectation values for fluxes in a compact geometry wrapped on a cycle, which leads to a potential energy that depends on the geometry of the cycle. This can be done by adding non-trivial harmonic p-forms to F_p. A harmonic p-form q_p^h satisfies by definition

$$dq_p^h = d\left(*q_p^h\right) = 0 \ , \tag{3.13}$$

so that the equations of motion are equally satisfied for F_p and $F'_p = F_p + q_p^h$. However, these harmonic forms q_p^h cannot be arbitrary since the fluxes F_p, F'_p must be integrally quantized due to Dirac quantization. Considering a p-cycle Σ_p in the internal space Y_6, this means that

$$\int_{\Sigma_p} F_p \propto N, \quad N \in \mathbb{Z} \ . \tag{3.14}$$

In the case in which we turn on the flux H_3 in (3.2), which is a common part of the action in Type IIA and IIB string theories, we generate a priori a potential of the form

$$V_F \propto \int d^{10}x\sqrt{-G_E}\ g^{mr}\cdots g^{ns}H_{m\dots n}H^{r\dots s} \ , \tag{3.15}$$

which connects with the moduli due to the dependence of the internal metric g_{mn} on them. These fluxes do not alter the conformal Calabi-Yau character of the compact manifold, but the internal space becomes more rigid and its continuous deformations now have an energetic cost. In this way the moduli obtain a potential. All this is true for Kähler, complex structure moduli and also the axio-dilaton τ defined in (3.8), in type IIA string theory, but in type IIB Kähler moduli are not stabilized by fluxes and one has to consider non-perturbative contributions to the action in order to generate a mass term for them. In the remainder of this work we assume that all moduli mentioned above are stabilized at sufficiently high mass scales.

3.3 Conditions on the fluxes

Flux compactifications depend crucially on the background of the theory. Actually, an arbitrary background geometry does not lead to non-vanishing vacuum expectation values for certain tensor fields in general. In vacuum configurations there are no sources of stress-energy, so one can consider the geometry

$$ds^2 = G_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu + g_{uv} dx^u dx^v \ , \tag{3.16}$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric and g_{uv} corresponds to the flat metric of the internal space. However, background fields can source stress-energy so this geometry is no longer valid. Demanding maximal symmetry in $\mathcal{M}_{1,3}$ with a general metric $g_{\mu\nu}$, what we get is a warped geometry of the form

$$ds^2 = G_{MN} dx^M dx^N = Z(x_{Y_6})^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + Z(x_{Y_6})^{1/2} g_{uv} dx^u dx^v \ , \tag{3.17}$$

where Z is a warp factor that depends only on the internal space coordinates due to Poincaré invariance, and g_{uv} does not have to be Ricci-flat. However, we will only deal with the Ricci-flat case in both internal and external parts. This warp factor has a non-trivial dependence on the fluxes through the equations of motion of the action (3.10). Before discussing this, it is very useful to use Poincaré invariance of $\mathcal{M}_{1,3}$ and the Bianchi identities to restrict the possible values of the fluxes G_3 and $\tilde{F}_5$. Firstly, we have that G_3 can only have non-zero components in the internal space. Otherwise it introduces a preferred direction that breaks Poincaré invariance in four dimensions. Secondly, the most general form of $\tilde{F}_5$ that respects both conditions is given by

$$\tilde{F}_5 = (1 + *_{10}) \ d\alpha(x_{Y_6}) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \ , \tag{3.18}$$

where α is a general function of the internal space. The Bianchi identity is trivial since $dd\alpha = 0$. Moving on from there, the equation that interests us is the Einstein equation derived from (3.10) for the $\mathcal{M}_{1,3}$ part

$$R_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} - \frac{1}{8} G_{\mu\nu} T \right) \ , \tag{3.19}$$

where $T_{\mu\nu}$ is the energy momentum tensor, defined by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-G_E}} \frac{\delta S_{IIB}}{\delta G_E^{\mu\nu}} \ , \tag{3.20}$$

and T is its trace. Computing both of them for the action (3.10) one obtains

$$R_{\mu\nu} = -\frac{1}{4} \eta_{\mu\nu} \left(\frac{1}{2 \ \mathrm{Im}[\tau]} |G_3|^2 + Z^2 |\partial\alpha|^2 \right) \ , \tag{3.21}$$

and writing the Ricci tensor in terms of the warp factor, one finally arrives at the equation

$$\nabla^2 Z^{-1} = \frac{Z^{-2}}{2 \ \mathrm{Im}[\tau]} |G_3|^2 + Z \left(|\partial\alpha|^2 + |\partial Z^{-1}|^2 \right) \ . \tag{3.22}$$

Integrating both sides of this equation over the internal space one obtains a no-go theorem. The left side of the equation vanishes since it is a total derivative, and the right side is a sum of positive definite terms. This equation of motion is only satisfied if Z and α are constant and the flux G_3 vanishes. Therefore, having a non-vanishing background flux G_3 is not as trivial as expected, and it is the flux that is useful for our purposes as we will see later on. A possible solution consists in introducing Dp-branes, which are explained in detail in the next section, wrapping $(p-3)$-cycles of the internal space. These local sources introduce negative terms on the right side of (3.22) through their local energy-momentum tensors, and lead to a more intricate equation than (3.22) from which we can extract two important constraints [18]. Firstly, the G_3 flux has to be imaginary self-dual (ISD), that is

$$*_6\, G_3 = iG_3 \ . \tag{3.23}$$

Secondly, the warp factor Z and the function α are related by

$$\alpha = Z^{-1} \ . \tag{3.24}$$

We must consider such Dp-brane local sources in our background and a non-vanishing $\tilde{F}_5$ form to source the warp factor Z and to construct a consistent flux compactification.

3.4 D-Branes, DBI and CS actions

There are other fundamental objects of great relevance in string theory in addition to strings. The ones we are going to consider are called "Dp-Branes". They are $(p+1)$-dimensional hypersurfaces to which open strings can be attached. They appear when $9-p$ Dirichlet boundary conditions are chosen for open strings. These conditions imply that the ends of the open string cannot move in the directions x^λ, with $\lambda = p+1, \ldots, 9$, defining such a hypersurface. The tensors defined in its worldvolume will have indices $a, b = 0, 1, \ldots, p$. Obviously, the dynamics of the D-branes are closely related to the ones of the open strings ending on them. In the massless content of its spectrum there are scalar fields associated with the fluctuations of the D-brane position, a $U(1)$ gauge field A_a with field strength F_{ab} and the corresponding superpartners. The latter ones will not be important for our purposes. In the supergravity limit, the coupling of a Dp-brane to this massless boson sector is given by the Dirac-Born-Infeld effective action. Considering a general closed-string background, this action in the 10d Einstein frame is given by

$$S_{DBI} = -T_p \int d^{p+1}\xi \, \sqrt{-\det\left(g_s^{1/2} G_{ab} + \mathcal{F}_{ab}\right)} \ . \tag{3.25}$$

Here ξ_a are the coordinates of the D-brane worldvolume, T_p is the brane tension defined by

$$T_p \equiv \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}} \ , \tag{3.26}$$

and $\mathcal{F}_{ab}$ is a gauge-invariant field strength given by

$$\mathcal{F}_{ab} \equiv -B_{ab} + 2\pi\alpha' F_{ab} \ . \tag{3.27}$$

The forms G_{MN} and B_{MN} are defined in the 10d target space-time, so the components G_{ab} and B_{ab} are obtained using the pullback into the worldvolume

$$G_{ab} \equiv P[G]_{ab} = G_{MN} \frac{\partial x^M}{\partial \xi^a} \frac{\partial x^N}{\partial \xi^b} \ . \tag{3.28}$$

The pullback exhibits the geometric interpretation of the DBI action. It is nothing but the proper volume swept out by the Dp-brane, analogous to the tail that a particle leaves behind in its trajectory. The kinetic terms for the different scalars are expected to appear at leading order in its expansion. This is why Dp-branes can be interpreted as dynamical objects. In addition, Dp-branes can also interact with the R-R sector of closed strings, which gives them an R-R charge. Actually, a Dp-brane is charged under C_{p+1} via the electric coupling

$$S_{CS} = \mu_p \int_{\Sigma_{p+1}} C_{p+1} \ , \tag{3.29}$$

with $\mu_p = g_s T_p$ the brane charge and Σ_{p+1} the brane worldvolume. The action in (3.29) is called the Chern-Simons action. It is a $(p+1)$-dimensional generalization of the action for a charged point particle coupled to a gauge potential. Considering a general background field, this action takes the form

$$S_{CS} = \mu_p \int_{\Sigma_{p+1}} \sum_n C_n \wedge e^{\mathcal{F}} \ . \tag{3.30}$$

The sum runs over all the possible C_p potentials, and only $(p+1)$-forms contribute to the worldvolume integral. Considering more than one D-brane, the worldvolume gauge theory is non-Abelian and the action is more complicated. We will deal only with the Abelian case, so all the dynamics of a D-brane are given by the sum of the actions (3.25) and (3.30).

3.5 The effective theory perspective

Although string theory could provide the ultraviolet completion that we need for inflation, it is a very intricate theory. What is usually done in order to avoid a large amount of difficult computations is to use string theory in the context of effective field theory, so inflation is derived in its low-energy limit. This approach is valid if the inflationary vacuum energy is much lower than the characteristic mass scale of the strings, M_s. The crucial point when an effective theory is used is to identify the light and heavy degrees of freedom. Introducing a cut-off energy Λ, all the light particles with masses $m < \Lambda$ are included in the effective theory while the heavy ones are integrated out through a path integral. In the low-energy limit $E \ll \Lambda < M$, the effective Lagrangian that arises from integrating out a heavy field with mass M admits a series expansion in powers of the ratio E/M as the following one

$$\mathcal{L}_{eff}[\phi] = \mathcal{L}_l[\phi] + \sum_i c_i(g) \frac{O_i[\phi]}{M^{\delta_i - 4}} \ , \tag{3.31}$$

where the dependence on the light fields ϕ is shown explicitely, $\mathcal{L}_l$ is the renormalizable part of the Lagrangian, $c_i(g)$ are dimensionless coefficients that depend on the couplings of the total theory and O_i are local operators of dimension δ_i. Therefore, in the limit in which the couplings g of the ultraviolet theory are very small, or in the limit in which the typical energy scale E is very small compared with the heavy masses M, there is a decoupling from the ultraviolet physics. Their possible contributions are effectively suppressed.

Moving on from here, the values for the energy scales and the couplings chosen are fundamental to validate the results of our inflationary model. The typical scale of energies that we have at inflation is identified with the value of the Hubble parameter at τ_*, that is

$$E \equiv H(\phi_*) \ . \tag{3.32}$$

Starting from the string scale M_s, it has to be necessarily lower than the Planck scale M_P to justify the non-renormalizability of quantum gravity. It also has to be large compared to E to accept that string excitations are negligible during inflation, so

$$M_P > M_s \gg H(\phi_*) \ . \tag{3.33}$$

For the last assumption it is also important to have a small string coupling g_s. This allows for a perturbative treatment and also avoids loop corrections that could increase the light scalar masses. Another important scale is the Kaluza-Klein energy scale M_{KK}. It is the typical energy of the tower of string excitation modes, and it has to be larger than E to reach the supergravity limit considered in section 3.1,

$$M_s > M_{KK} \gg H(\phi_*) \ . \tag{3.34}$$

This allows us to consider only the massless sectors of string theory. Moreover, this energy scale can be identified with the volume V_6 of the compactification as

$$M_{KK} \sim V_6^{-1/6} \ . \tag{3.35}$$

Finally, the masses of the different moduli that appear in string theory have to be larger than E and the mass of the inflaton to accept that all their physical interactions are effectively suppressed.

$$m_{\text{Moduli}} \gg H(\phi_*), \ m_\phi \ . \tag{3.36}$$

4 An inflationary model

In the context of string theory, we have discussed that there are many moduli that have to receive a mass through mechanisms like flux compactifications. There are simple cases in which we can identify one of these moduli with the inflaton, and impose a concrete background to obtain results that reproduce the cosmological observables. In this work we study the moduli related with the position of a D7-brane in the internal space reproducing the computations of [10]. In that reference a constant warp factor $Z = 1$ is considered, and in this work we will extend the computations to the case in which this factor has a periodical dependence on the inflaton field. First we reproduce the computations without assuming anything about Z, and later on we will study the consequences that appear when it is not constant.

Starting with the geometry of the system, we consider a 10-dimensional flat space-time of the form

$$\mathcal{M} = \mathcal{M}_{1,3} \times \mathbf{T}^4 \times \mathbf{T}^2 \ , \tag{4.1}$$

where $\mathcal{M}_{1,3}$ is the usual 4-dimensional space-time with a flat FRW metric $g_{\mu\nu}$ given by (2.1), and we have an internal complex compact space given by the product of a 2-torus and a 4-torus. The internal coordinates can be identified with the set of complex variables $\{z_1, z_2, z_3\}$ and its complex conjugates, using $\{z_3, \bar{z}_3\}$ for the coordinates of $\mathbf{T}^2$ and the other ones for $\mathbf{T}^4$. z_3 is the candidate direction for the inflaton. As for the closed-string background, we consider a Ricci-flat warped geometry of the form (3.17) with a warp factor Z that can depend only on $\{z_3, \bar{z}_3\}$ and that is sourced by a $\tilde{F}_5$ form given by

$$\tilde{F}_5 = (1 + *_{10})\, dZ^{-1} \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \ , \tag{4.2}$$

and also a 10d axio-dilaton τ defined in (3.8) and an imaginary self-dual (ISD) primitive flux G_3 of the form

$$\begin{aligned} G_3 = S_{\bar{1}\bar{1}} d\bar{z}_1 \wedge dz_2 \wedge dz_3 + S_{\bar{2}\bar{2}} dz_1 \wedge d\bar{z}_2 \wedge dz_3 + \\ S_{\bar{3}\bar{3}} dz_1 \wedge dz_2 \wedge d\bar{z}_3 + G_{\bar{1}\bar{2}\bar{3}} d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3 \ , \end{aligned} \tag{4.3}$$

that is related to the R-R and NS-NS fluxes through (3.9). In this way, the conditions discussed in section 3.3 are satisfied and we have a consistent flux compactification. We are just assuming that we have the necessary local sources to satisfy the equations of motion, but that they do not have any other significant nteraction with the system. Finally, we consider a D7-brane whose worldvolume covers the $\mathcal{M}_{1,3}$ and $\mathbf{T}^4$ manifolds, so it is free to move along $\mathbf{T}^2$. Its dynamics will be studied from the DBI and the CS actions discussed in section 3.4.

Because of the structure of the manifold, it results that the different tensor fields do not have crossed terms between the $\mathcal{M}_{1,3}$ and the internal parts. We also assume that $B_{\mu\nu} = 0$ since we do not observe such a field in 4d. Finally, to simplify computations we consider that the field strength F is non-zero only in the 4d components, and that $B_{ij} = 0$. We will use the indices i, j for the $\mathbf{T}^2$ and the indices m, n for the $\mathbf{T}^4$. Once the fundamental ingredients of the model have been defined, the next step is to obtain a 4d effective action that allows us to identify one of the scalar fields with an inflaton. Note that the results of this thesis are quite likely more general than the simple geometry we have chosen in (4.1). (see section 3 of [10]).

4.1 Dimensional reduction of the DBI action

We start from the DBI action given by[8]

$$S_{DBI} = -\mu_7 g_s^{-1} \left(\int d^8\xi \, \sqrt{-\det\left(P[E]_{ab} + \sigma F_{ab}\right)} \right) , \tag{4.4}$$

with

$$E_{MN} = g_s^{1/2} G_{MN} - B_{MN} \ . \tag{4.5}$$

In what follows it is better to work with real coordinates for the internal space instead of the complex ones. They are just related by

$$z_1 = \frac{1}{\sqrt{2}} (x_4 + i x_5) \ , \qquad z_2 = \frac{1}{\sqrt{2}} (x_6 + i x_7) \ , \qquad z_3 = \frac{1}{\sqrt{2}} (x_8 + i x_9) \ . \tag{4.6}$$

Regarding the transverse displacements of the D7-brane, the following parametrization will be used

$$x_i(x_\mu) = 2\pi\alpha' y_i(x_\mu) \equiv \sigma y_i(x_\mu) \ . \tag{4.7}$$

We assume that these two coordinates can only depend on $\mathcal{M}_{1,3}$ coordinates. First of all, we have to do the pullback of the tensors living on the 10-dimensional target space-time

$$P[E]_{ab} = E_{ab} + \sigma E_{aj} \partial_b y^j + \sigma E_{ib} \partial_a y^i + \sigma^2 E_{ij} \partial_a y^i \partial_b y^j \ . \tag{4.8}$$

The determinant of (4.4) can be factorized between $\mathcal{M}_{1,3}$ and internal space. Therefore, the argument of the determinant can be separated in the following pieces[9]

$$P[E]_{ab} + \sigma F_{ab} = N_{\mu\nu} + M_{mn} + X_{\mu n} + Y_{m\nu} \ , \tag{4.9}$$

- $\mathcal{M}_{1,3}$ part

$$N_{\mu\nu} = g_s^{1/2} Z^{-1/2} \left(\eta_{\mu\nu} + \sigma^2 Z g_{ij} \partial_\mu y^i \partial_\nu y^j + g_s^{-1/2} Z^{1/2} \sigma F_{\mu\nu} \right) \ . \tag{4.10}$$

- Internal part

$$M_{mn} = g_s^{1/2} Z^{1/2} \left(g_{mn} + g_s^{-1/2} Z^{-1/2} \mathcal{F}_{mn} \right) \ . \tag{4.11}$$

- Crossed parts

$$X_{\mu n} = E_{\mu n} + \sigma E_{\mu j} \partial_n y^j + \sigma E_{in} \partial_\mu y^i + \sigma^2 E_{ij} \partial_\mu y^i \partial_n y^j = 0 \ , \tag{4.12}$$
$$Y_{m\nu} = E_{m\nu} + \sigma E_{mj} \partial_\nu y^j + \sigma E_{i\nu} \partial_m y^i + \sigma^2 E_{ij} \partial_m y^i \partial_\nu y^j = 0 \ . \tag{4.13}$$

Using the following determinant formula

$$\det \begin{pmatrix} N & -A^T \\ A & M \end{pmatrix} = \det(N) \det\left(M + A N^{-1} A^T\right) \ , \tag{4.14}$$

we obtain the desired factorization. Since our cross terms are zero, it simply results in

$$\det\left(P[E]_{ab} + \sigma F_{ab}\right) = g_s^4 \det\left(\eta_{\mu\nu} + 2\sigma^2 \partial_\mu \Phi \partial_\nu \bar{\Phi} + Z^{1/2} g_s^{-1/2} \sigma F_{\mu\nu}\right) \tag{4.15}$$
$$\cdot \det\left(g_{mn} + Z^{-1/2} g_s^{-1/2} \mathcal{F}_{mn}\right) \ , \tag{4.16}$$

[8]Note that it is just a redefinition of (3.25) in which we emphasize the tensor fields that need a pullback.

[9]Note that each part can be viewed as a 4x4 matrix.

where in the first determinant we have changed to complex coordinates $\Phi \equiv (y^8 + iy^9)/\sqrt{2}$ so[10] $g_{ij}\partial_\mu y^i \partial_\nu y^j \to 2\partial_\mu \Phi \partial_\nu \bar{\Phi}$. The $\mathcal{M}_{1,3}$ determinant can be evaluated using the expansion

$$\det\left(\mathbb{I} + \epsilon M\right) = 1 + \epsilon \text{Tr } M - \frac{\epsilon^2}{2}\left[\text{Tr } M^2 - (\text{Tr } M)^2\right] + O(\epsilon^3) \ . \tag{4.17}$$

Since we want to work at an energy scale H much lower than the string scale M_s, all the terms of order σ^3 and higher can be ignored. Replacing $\eta_{\mu\nu}$ by $\delta_{\mu\nu}$ yields a relative factor (-1), simply using that $\det(AB) = \det(A)\det(B)$. Therefore, keeping only the leading-order terms we get for the first determinant

$$\det{}_{\mathcal{M}_{1,3}} = -\left(1 + 2Z\sigma^2\left[\partial_\mu \Phi \partial^\mu \bar{\Phi} + \frac{1}{4g_s} F_{\mu\nu}F^{\mu\nu}\right]\right) + O(\sigma^3) \ . \tag{4.18}$$

As for the second determinant, because of the antisymmetry of $\mathcal{F}$ we have that $\text{Tr}(\mathcal{F}) = 0$. This implies the identity

$$\det(1 + \epsilon\mathcal{F}) = 1 + \epsilon^2 \mathcal{F}^2 + \frac{1}{4}\epsilon^4 \left(\mathcal{F}\wedge\mathcal{F}\right)^2 \ , \tag{4.19}$$

which leads to the result

$$\det(g_{mn}) \det\left(\mathbb{I} + Z^{-1/2} g_s^{-1/2} \mathcal{F}\right) = f(\mathcal{F}) \ , \tag{4.20}$$

where

$$f(\mathcal{F}) = 1 + g_s^{-1} Z^{-1} \mathcal{F}^2 + \frac{1}{4} g_s^{-2} Z^{-2} \left(\mathcal{F}\wedge\mathcal{F}\right)^2 \ , \tag{4.21}$$

and the square of a p-form is defined as

$$\omega_p^2 = \frac{1}{p!}\omega_{a_1 \dots a_p}\omega^{a_1 \dots a_p} \ . \tag{4.22}$$

To sum up, factorizing the $\mathcal{M}_{1,3}$ and the internal space parts and keeping only the terms least suppressed by M_s we get the following DBI action

$$S_{DBI} = -\mu_7 g_s \int d^8\xi \sqrt{f(\mathcal{F})\left(1 + 2Z\sigma^2 \partial_\mu \Phi \partial^\mu \bar{\Phi} + \frac{Z}{2g_s}\sigma^2 F_{\mu\nu}F^{\mu\nu}\right)} \ . \tag{4.23}$$

Since nothing depends on the coordinates of the internal space, the 4d effective action is proportional to the volume V_4 of the internal $\mathbf{T}^4$. The square root can be expanded in powers of σ^2, so terms of the order σ^4 and higher can be ignored again. Therefore, the resulting DBI action is

$$S_{DBI} = -\mu_7 g_s V_4 \int \text{dvol}_{\mathcal{M}_{1,3}} \sqrt{f(\mathcal{F})}\left(1 + Z\sigma^2 \partial_\mu \Phi \partial^\mu \bar{\Phi} + \frac{1}{4g_s} Z\sigma^2 F_{\mu\nu}F^{\mu\nu}\right) \ . \tag{4.24}$$

The only thing that remains to be specified is the form of the function $f(\mathcal{F})$. The 2-form B_2 satisfies by definition $dB_2 = H_3$, and this relation can be rewritten in terms of the G_3 flux and the axio-dilaton τ in the following way

$$dB_2 = -\frac{\text{Im}[G_3]}{\text{Im}[\tau]} = \frac{g_s}{2i}\left(\bar{G}_3 - G_3\right) \ . \tag{4.25}$$

[10]Remember that we are assuming a flat internal space, so $g_{ij} = \delta_{ij}$.

We can integrate this relation to obtain

$$B_2 = \frac{g_s\sigma}{2i}\left(\bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi - S_{\bar{3}\bar{3}}\bar{\Phi}\right) dz_1 \wedge dz_2 + \frac{g_s\sigma}{2i}\left(\bar{S}_{\bar{1}\bar{1}}\bar{\Phi} - S_{\bar{2}\bar{2}}\Phi\right) dz_1 \wedge d\bar{z}_2 + \text{h.c.} \ . \tag{4.26}$$

Therefore, the function $f(\mathcal{F})$ is given by

$$f(\Phi, \bar{\Phi}) = 1 + \frac{g_s\sigma^2}{4Z}(\mathcal{G} + \mathcal{H}) + \frac{1}{4}\left(\frac{g_s\sigma^2}{4Z}\right)^2 (\mathcal{G} - \mathcal{H})^2 \ , \tag{4.27}$$

where we have defined the variables

$$\mathcal{G} \equiv |\bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi - S_{\bar{3}\bar{3}}\bar{\Phi}|^2 \ , \qquad \mathcal{H} \equiv |\bar{S}_{\bar{1}\bar{1}}\bar{\Phi} - S_{\bar{2}\bar{2}}\Phi|^2 \ . \tag{4.28}$$

4.2 Dimensional reduction of the CS action

As mentioned before, the dynamics of the D7-brane also depends on the Chern-Simons action. We start from the expression (3.30). Expanding the exponential $e^{\mathcal{F}}$, the terms that contribute to the integral are

$$S_{CS} = \mu_7 \int (C_8 - C_6 \wedge B_2) \ . \tag{4.29}$$

The p-forms C_6 and C_8 appear when the roles of the field strengths $F_1 \leftrightarrow *F_1$ and $F_3 \leftrightarrow *F_3$ are interchanged. It is just a redistribution of the dual degrees of freedom, but in order to have consistent equations of motion one has to apply the relations

$$dC_6 = H_3 \wedge C_4 - g_s *_{10} \text{Re}\, G_3 \ , \tag{4.30}$$

$$dC_8 = H_3 \wedge C_6 - g_s^2 *_{10} \text{Re}\, d\tau \ . \tag{4.31}$$

These relations are useful since we can integrate them to find C_6 and C_8 in terms of the components of the 2-form B_2. In this way we find

$$S_{CS} = \mu_7 V_4 \int \text{dvol}_{\mathcal{M}_{1,3}} \left(-\frac{1}{4} B_2 \wedge B_2 + g_s\right) \ . \tag{4.32}$$

The term g_s is included by hand, and the reason is that the potential term has to be zero by construction when the fluxes are turned off. Computing the wedge product and using the definitions (4.28), one obtains the following contribution from the CS action

$$S_{CS} = -\mu_7 g_s V_4 \int \text{dvol}_{\mathcal{M}_{1,3}} \left(\frac{1}{2}\frac{g_s\sigma^2}{4Z}(\mathcal{G} - \mathcal{H}) - 1\right) \ . \tag{4.33}$$

4.3 The scalar potential

Adding the contributions of (4.24) and (4.33) we finally get the following 4d effective action

$$S_{4d} = -\int \mathrm{dvol}_{\mathcal{M}_{1,3}} \left(\chi(\Phi, \bar{\Phi}) \partial_\mu \Phi \partial^\mu \bar{\Phi} + V(\Phi, \bar{\Phi}) + \frac{1}{4 g_{YM}^2} F_{\mu\nu} F^{\mu\nu} \right) , \tag{4.34}$$

where we have defined

$$g_{YM}^{-2} = \mu_7 V_4 Z \sigma^2 \sqrt{f(\Phi, \bar{\Phi})} , \tag{4.35}$$

$$\chi(\Phi, \bar{\Phi}) = \mu_7 g_s V_4 Z \sigma^2 \sqrt{f(\Phi, \bar{\Phi})} , \tag{4.36}$$

$$V(\Phi, \bar{\Phi}) = \mu_7 g_s V_4 \left(\sqrt{f(\Phi, \bar{\Phi})} + \frac{1}{2} \frac{g_s \sigma^2}{4Z} (\mathcal{G} - \mathcal{H}) - 1 \right) . \tag{4.37}$$

In the following, we will focus only on the kinetic term and the potential $V(\Phi, \bar{\Phi})$, since they are the important ones for inflation. Here we can see why a non-zero value of G_3 is needed, something commented in section 3.3. If $|G_3|^2 = 0$, then we have that

$$|S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2 + |S_{\bar{3}\bar{3}}|^2 + |G_{\bar{1}\bar{2}\bar{3}}|^2 = 0 . \tag{4.38}$$

Since this is a sum of positive definite terms, it implies that all terms would have to vanish. Inserting this result in (4.27) a trivial function $f(\Phi, \bar{\Phi}) = 1$ is obtained, so the potential (4.37) is actually zero. Therefore, we need to satisfy the condition $|G_3|^2 \neq 0$ in order to achieve an inflationary potential.

The action (4.34) depends on a lot of constant parameters. We perform a field redefinition in order to simplify the coming computations

$$\Phi \longrightarrow \Phi \left(\mu_7 g_s V_4 \sigma^2 \right)^{-1/2} , \tag{4.39}$$

so the kinetic and potential terms become

$$\chi(\Phi, \bar{\Phi}) = Z \sqrt{1 + \hat{\epsilon}(\mathcal{G} + \mathcal{H}) + \frac{1}{4} \hat{\epsilon}^2 (\mathcal{G} - \mathcal{H})^2} , \tag{4.40}$$

$$V(\Phi, \bar{\Phi}) = V_0 \left(\sqrt{1 + \hat{\epsilon}(\mathcal{G} + \mathcal{H}) + \frac{1}{4} \hat{\epsilon}^2 (\mathcal{G} - \mathcal{H})^2} + \frac{1}{2} \hat{\epsilon} (\mathcal{G} - \mathcal{H}) - 1 \right) , \tag{4.41}$$

$$\hat{\epsilon} \equiv \frac{1}{4 \mu_7 V_4 Z} , \tag{4.42}$$

$$V_0 \equiv \mu_7 g_s V_4 . \tag{4.43}$$

Looking at the potential (4.41) we see that the fluxes guarantee a non-vanishing mass term for the fields $\Phi, \bar{\Phi}$. Expanding around $\Phi = 0$ to second order such mass terms come out. Since $\mathcal{G}, \mathcal{H} \propto |\Phi|^2$, we first expand around $\mathcal{G} = 0$, finding

$$V(\Phi, \bar{\Phi}) = V_0 \left(\frac{2\hat{\epsilon}}{2 + \hat{\epsilon}\mathcal{H}} \mathcal{G} + \frac{2\hat{\epsilon}^3 \mathcal{H}}{(2 + \hat{\epsilon}\mathcal{H})^3} \mathcal{G}^2 \right) + O(\mathcal{G}^3) , \tag{4.44}$$

and after that we expand around $\mathcal{H} = 0$, obtaining

$$V(\Phi, \bar{\Phi}) = V_0 \hat{\epsilon} \mathcal{G} + O(\mathcal{G}^2, \mathcal{H}^2, \mathcal{G}\mathcal{H}) . \tag{4.45}$$

Therefore the flux components $S_{\bar{1}\bar{1}}$, $S_{\bar{2}\bar{2}}$ do not contribute to the mass of Φ and $\bar{\Phi}$. Since there is a dependence on the warp factor Z through $\hat{\epsilon}$, one can consider the possibility that a non-constant warp factor affects the mass terms, but here it is not the case. Because of the relation $\mathcal{G} \propto |\Phi|^2$, only the constant part of Z is inside the mass terms, and the non-constant part creates potential terms of order $O(|\Phi|^3)$ and higher. Expanding $\mathcal{G}$ in terms of the real and the imaginary parts of the field Φ leads to

$$\mathcal{G} = \left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2\right)|\Phi|^2 - 2\mathrm{Re}\left(\bar{G}_{\bar{1}\bar{2}\bar{3}}\bar{S}_{\bar{3}\bar{3}}\Phi^2\right) . \tag{4.46}$$

To obtain ordinary mass terms we have to do the field redefinition

$$\Phi' = e^{-i\gamma/2}\Phi , \qquad \gamma \equiv \mathrm{Arg}(G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}}) . \tag{4.47}$$

In this way, one finally obtains the following mass terms

$$\begin{aligned}\frac{V(\Phi',\bar{\Phi}')}{V_0} &= \hat{\epsilon}\left(|G_{\bar{1}\bar{2}\bar{3}}| - |S_{\bar{3}\bar{3}}|\right)^2 \mathrm{Re}\left(\Phi'\right)^2 + \\ &\quad \hat{\epsilon}\left(|G_{\bar{1}\bar{2}\bar{3}}| + |S_{\bar{3}\bar{3}}|\right)^2 \mathrm{Im}\left(\Phi'\right)^2 + O(\mathcal{G}^2,\mathcal{H}^2,\mathcal{G}\mathcal{H}) ,\end{aligned} \tag{4.48}$$

for the fields $\mathrm{Re}\left(\Phi'\right), \mathrm{Im}\left(\Phi'\right)$. When $|G_{\bar{1}\bar{2}\bar{3}}| \simeq |S_{\bar{3}\bar{3}}|$, the field $\mathrm{Re}\left(\Phi'\right)$ can be very light compared to $\mathrm{Im}\left(\Phi'\right)$. This allows us to consider $\mathrm{Im}\left(\Phi'\right)$ to be frozen at the origin and to identify $\phi \equiv \mathrm{Re}\left(\Phi'\right)$ as the inflaton field. Moreover, making the assumption that

$$\mathrm{Arg}(S_{\bar{1}\bar{1}}S_{\bar{2}\bar{2}}) = -\mathrm{Arg}(G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}}) , \tag{4.49}$$

we can repeat this procedure for $\mathcal{H}$, obtaining

$$\mathcal{H} = \left(|S_{\bar{2}\bar{2}}| - |S_{\bar{1}\bar{1}}|\right)^2 \mathrm{Re}\left(\Phi'\right)^2 + \left(|S_{\bar{2}\bar{2}}| + |S_{\bar{1}\bar{1}}|\right)^2 \mathrm{Im}\left(\Phi'\right)^2 . \tag{4.50}$$

It will be needed in section 4.5 to obtain the potential (4.61).

4.4 A non-constant warp factor

The novelty that we are going to introduce with respect to the results of [10] is a non-constant warp factor $Z(\phi)$ that has a periodical dependence on the inflaton field. The warp factor is a globally well-defined function of the internal space Y_6, which is compact. Because we are identifying the inflaton field with the position of a D7-brane on $\mathbf{T}^2$, when it completes a loop the warp factor has the same value as when it started. Then Z is allowed to have, by construction, a periodical dependence on the coordinates of $\mathbf{T}^2$. Considering the periodicity

$$Z\left(x_8 + a, x_9\right) = Z\left(x_8, x_9 + b\right) = Z\left(x_8, x_9\right) , \tag{4.51}$$

the warp factor can be expanded in a Fourier decomposition of the form

$$Z\left(x_8, x_9\right) = \sum_{n,m=-\infty}^{\infty} Z_{nm} \exp\left(2\pi i\left(\frac{n}{a}x_8 + \frac{m}{b}x_9\right)\right) , \qquad n,m \in \mathbb{Z} . \tag{4.52}$$

Here we rewrite the dependence on x_8, x_9 in terms of $\mathrm{Re}\left(\Phi'\right), \mathrm{Im}\left(\Phi'\right)$ following all the definitions of the last sections, and ignore the imaginary part since it will be considered to be frozen at the origin. The resulting expression is given by

$$Z\left(\phi\right) = \sum_{n,m=-\infty}^{\infty} Z_{nm} \exp\left(2\pi i\left[\frac{\sigma\cos\left(\gamma/2\right)}{\sqrt{2}a}n + \frac{\sigma\sin\left(\gamma/2\right)}{\sqrt{2}b}m\right]\phi\right) . \tag{4.53}$$

Therefore, identifying

$$a = \frac{\sigma\cos(\gamma/2)}{\sqrt{2}} \ , \qquad\qquad b = \frac{\sigma\sin(\gamma/2)}{\sqrt{2}} \ , \tag{4.54}$$

one can show that for any background flux the periodic warp factor can be expressed in the simple form

$$Z(\phi) = \sum_{n,m=-\infty}^{\infty} Z_{nm}\exp\left(2\pi i\,[n+m]\,\phi\right) \equiv \sum_{n=-\infty}^{\infty} Z_n \exp\left(2\pi i n\phi\right) \ . \tag{4.55}$$

The introduction of such a warp factor can yield many different consequences. Fast oscillating modes propagate in the kinetic and potential terms, potentially spoiling the slow-roll regime during inflation. Such oscillations can also propagate to the cosmological observables, driving them out of the experimental bounds. The aim of this work is to find constraints on the form of $Z(\phi)$ using cosmology experimental data. We will study the simple case in which

$$Z(\phi) = Z_0\Big(1 + k\cos\left(2\pi F\phi\right)\Big) \equiv Z_0\Big(1 + J(\phi)\Big) \ , \tag{4.56}$$

to extract the general features and to make it easier to compare our results to those of [10], where $Z_0 = 1$ and $J(\phi) = 0$. This constant value is related to the size of the internal space through (3.17). In addition to the freedom of choosing our vacuum with different fluxes and internal volumes, and then different Z_0, this constant factor can be reabsorved by the metric using a constant rescaling. This is why we can choose simply $Z_0 = 1$. Moreover, a necessary condition on $J(\phi)$ is that its absolute value has to be always less than 1. Otherwise, for some values of the inflaton field the warp factor could vanish and then the warped metric (3.17) is not well-defined.

4.5 Model for a single field

Following the discussion of the final part of 4.3, we consider a background in which $|G_{\bar{1}\bar{2}\bar{3}}| \simeq |S_{\bar{3}\bar{3}}|$ to achieve inflation with a single scalar field ϕ. It is useful to define the quantities

$$\hat{G} \equiv \hat{\epsilon}\left(|G_{\bar{1}\bar{2}\bar{3}}| - |S_{\bar{3}\bar{3}}|\right)^2 \ , \tag{4.57}$$

$$\Upsilon \equiv \frac{|S_{\bar{2}\bar{2}}|^2 + |S_{\bar{1}\bar{1}}|^2}{|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2} \ , \tag{4.58}$$

and also assume that the G_3 flux satisfies

$$\frac{2|S_{\bar{1}\bar{1}}S_{\bar{2}\bar{2}}|}{|S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2} = \frac{2|G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}}|}{|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2} \ . \tag{4.59}$$

Then, the kinetic and potential terms (4.40) and (4.41) become

$$\chi(\phi) = Z\sqrt{1 + (1+\Upsilon)\hat{G}\phi^2 + \frac{1}{4}(\Upsilon-1)^2\hat{G}^2\phi^4} \ , \tag{4.60}$$

$$\frac{V(\phi)}{V_0} = \sqrt{1 + (1+\Upsilon)\hat{G}\phi^2 + \frac{1}{4}(\Upsilon-1)^2\hat{G}^2\phi^4} + \frac{1}{2}(1-\Upsilon)\hat{G}\phi^2 - 1 \ . \tag{4.61}$$

Before doing any numerical evaluation of the system, we must determine which values of the parameters appearing in these terms respect the scales we have imposed in our model and lead to cosmological observables in agreement with experimental data. In type IIB compactifications we have the following identities [19]

$$M_s^8 V_6 = \pi \left(2\pi\right)^6 g_s M_P^2 \ , \tag{4.62}$$

$$M_{KK} = M_s \left(\frac{2\alpha_G}{g_s}\right)^{1/4} \ , \tag{4.63}$$

$$V_4 = \left(\frac{2\pi}{M_{KK}}\right)^4 \ , \tag{4.64}$$

where α_G is the gauge coupling on the brane and the internal volume V_6 can be factorized as $V_6 = V_4 \cdot V_2$, with V_2 the volume of $\mathbf{T}^2$. In our computations we will consider $g_s = 0.1$ and $\alpha_G = 1/25$, so

$$V_4 = \frac{\left(2\pi\right)^4 g_s}{2\alpha_G M_s^4} \simeq 2 \times 10^3 M_s^{-4} \ , \tag{4.65}$$

$$V_2 = \frac{(2\pi)^3 \alpha_G M_P^2}{M_s^4} \simeq 10 M_s^{-4} \ , \tag{4.66}$$

and $M_{KK} \simeq 0.95 M_s$. The amplitude of the potential, V_0, depends on V_4 through the relation (4.43), so we can rewrite it as

$$V_0 = \frac{g_s^2 M_s^4}{2(2\pi)^3 \alpha_G} \simeq 5 \times 10^{-4} M_s^4 \ . \tag{4.67}$$

Finally, the masses of the inflaton ϕ and the scalar field $\mathrm{Im}\,(\Phi')$ depend on V_0 and also on the flux-dependent parameters Υ and $\hat{G}_0$. We define $\hat{G}_0$ as the value[11] of $\hat{G}$ when $Z = Z_0$. Actually, we expect the mass of the axio-dilaton and the complex structure moduli to be of the order of $|S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2$ if those components of G_3 are the dominant ones. Considering that case, the parameter Υ relates the mass of these moduli with the one of $\mathrm{Im}\,(\Phi')$, so

$$\Upsilon \sim \frac{m^2_{\text{Moduli}}}{m^2_{\mathrm{Im}(\Phi')}} \ . \tag{4.68}$$

From (3.14) we expect the mass of the moduli to be

$$m_{\text{Moduli}} \sim \frac{\alpha'}{V_6^{1/2}} \simeq \frac{M_{KK}^3}{M_s^2} \ , \tag{4.69}$$

so looking at (4.48) we find that

$$m_\phi = \sqrt{2\hat{G}_0 V_0} \simeq 0.02\sqrt{\hat{G}_0} M_s^2 \ , \tag{4.70}$$

$$m_{\mathrm{Im}(\Phi')} \sim \left(\frac{8\alpha_G^3}{\Upsilon^2 g_s}\right)^{1/4} M_s \simeq 0.5\Upsilon^{-1/2} M_s \ . \tag{4.71}$$

Moving on from here, we have to determine the conditions that the set of parameters $(M_s, \hat{G}_0, \Upsilon)$ must fulfill. Firstly, in our computations we have ignored the axio-dilaton

[11] It is true that having a non-constant warp factor changes the situation, but because the absolute value of $J(\phi)$ is smaller than 1, the value of $\hat{G}$ is effectively of the same order of $\hat{G}_0$. Moreover, we will find in the next section tight constraints on the possible values of k that support this argument.

and the complex structure from the beginning, so their masses must be large enough to ignore them from the effective theory point of view. Therefore, it is necessary that $\Upsilon \gg 1$, but it cannot be arbitrarily large since it lowers the mass of the scalar field $\mathrm{Im}\,(\Phi')$. To fulfill the condition $m_{\mathrm{Im}(\Phi')} \gg H(\phi_*), m_\phi$, we will consider the typical range $\Upsilon \sim 10^2 - 10^3$. Secondly, the string scale M_s can be fixed making a choice of V_0. The slow-roll parameters are not expected to depend on the scale of the potential, so the amplitude V_0 has no effect on the cosmological observables r, n_s and its running. Actually, this property can be seen directly in the slow-roll limit through the relations (2.25)-(2.27), where the amplitude cancels, but it is also satisfied out of the slow-roll limit. However, the Hubble parameter depends on the scale of the potential, so the amplitude of the scalar perturbations A_s is the only observable related to V_0. The observable A_s can be computed through (2.50), a relation that becomes

$$A_s = \frac{V(\phi_*)}{24\pi^2 \epsilon_*} \,, \tag{4.72}$$

in the slow-roll limit, where we can see clearly such dependence. Therefore, V_0 and then M_s must be chosen to obtain a value of A_s that coincides with the measured one (2.51). After fixing M_s, it must be checked if the hierarchy of energies that it brings is the correct one. Finally, the $\hat{G}_0$ parameter presents a dependence on V_4 and the brane tension μ_7 through $\hat{\epsilon}$, but once they are fixed by M_s its value shows the fine-tuning of the relation $|G_{\bar{1}\bar{2}\bar{3}}| \simeq |S_{\bar{3}\bar{3}}|$. Since $\hat{G}_0$ affects the Hubble parameter and the mass of the inflaton, the only requirement is that it has to be small enough to keep them well below the other energy scales considered.

To summarize, once we have determined M_s using the observable A_s, we fix $\Upsilon \sim 10^2 - 10^3$ to guarantee that the masses of the two scalar fields we have considered are well below the mass of the other moduli. The parameter $\hat{G}_0$ must be small enough to respect the hierarchy of energies, but we will see in the next section that there are different values that can be chosen and that lead to different shapes of the inflationary potential.

5 Numerical evaluation of the model

5.1 Time evolution during inflation

Once the potential and kinetic terms are given, there are two ways of computing the cosmological observables discussed in 2.6. If the slow-roll conditions are satisfied, then the value of the inflaton at horizon crossing can be found without solving the time evolution of the Hubble parameter H and the canonically normalized inflaton[12] $\hat{\phi}$. Using (2.13) one determines the value of the inflaton at the end of inflation, $\hat{\phi_{end}}$, and the only thing that remains is specifying the number of e-folds that inflation lasts. In the slow-roll limit, the equation (2.12) can be rewritten in terms of the slow-roll parameters. One finds

$$N_e = \int_{\hat{\phi}_{end}}^{\hat{\phi}_*} d\hat{\phi} \sqrt{\frac{1}{2\epsilon(\hat{\phi})}} \,, \tag{5.1}$$

where we have considered the case of a non-canonical kinetic term. Therefore, imposing a value for N_e one can find numerically $\hat{\phi}_*$, and then evaluate the slow-roll parameters at horizon crossing to find the cosmological observables. Nevertheless, the slow-roll conditions are not always satisfied, and even less if we introduce a non-constant warp factor in the system. The correct way to proceed in such situations is to solve the system of differential equations (2.16)-(2.19) to find the time evolution of H and ϕ. It is difficult to obtain an analytic result for these equations in general, so one has to solve the system numerically using programs like *Mathematica*. This kind of evaluation needs a set of initial conditions, but the time evolution of H and ϕ during inflation does not depend on them. Equation (2.16) contains a friction term[13] that implies an attractor behavior for the inflationary evolution. Whatever the initial conditions are, after a transient regime the first derivative of the inflaton will reach a terminal negative velocity given by the

[12]We discussed in section 2.4 that it is convenient to work with $\hat{\phi}$ in the slow-roll limit.
[13]It is formed by the term with $\dot{\phi}$ and $\dot{\phi}^2$.

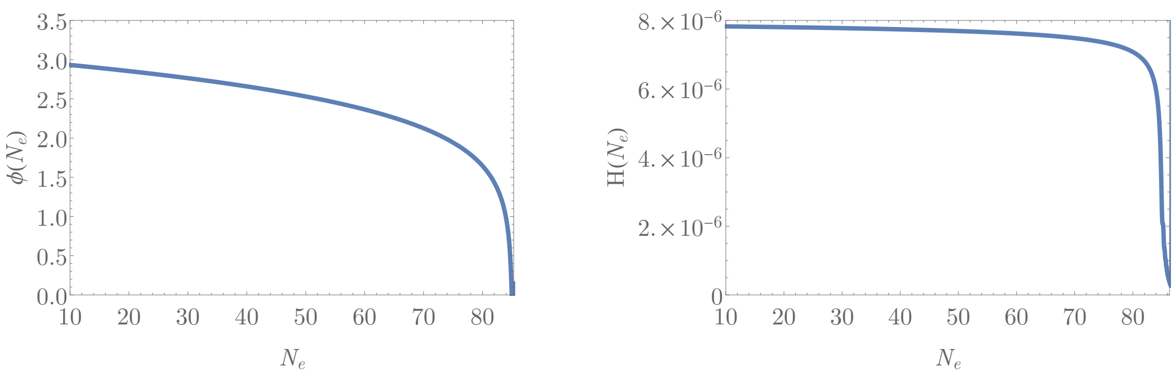

Figure 1: Time evolution of the inflaton field ϕ and the Hubble parameter H in terms of the number of e-folds for $\hat{G}_0 = 10^{-3}$ and $\Upsilon = 2 \cdot 10^3$. The value of V_0 is normalized to obtain the experimental scalar amplitude A_s considering 60 e-folds of inflation. Its value can be found in table 1.

second-order equation

$$\dot{\phi}^2 + \frac{6H\chi(\phi)}{\chi'(\phi)}\dot{\phi} + \frac{2V'(\phi)}{\chi'(\phi)} = 0 \ , \tag{5.2}$$

and from this result the velocity of H can be found through (2.19). Therefore, the initial conditions for ϕ, H and its derivatives have to be chosen in order to obtain an inflationary period that lasts over $50-60$ e-folds and that is far away from the initial transient regime of the trajectory. In figure 1 we present an example of the resulting time evolution for the inflaton field and the Hubble parameter. Once they are known, the slow-roll parameters can be computed during inflation through their definitions without using any approximation.

5.2 Analysis of the potential shapes

From section 4.5 we have gained some intuition about the possible values for the parameters of the potential (4.61). Rewriting the square root of this potential we find that there are two interesting regimes that can be achieved with such values. Using that

$$1 + (1+\Upsilon)\hat{G}\phi^2 + \frac{1}{4}\left(\Upsilon-1\right)^2\hat{G}^2\phi^4 = \left(1 + \frac{1}{2}\left(\Upsilon-1\right)\hat{G}\phi^2\right)^2 + 2\hat{G}\phi^2 \ , \tag{5.3}$$

the square root can be expanded in the limit in which

$$\frac{2\hat{G}\phi^2}{\left(1 + \frac{1}{2}\left(\Upsilon-1\right)\hat{G}\phi^2\right)^2} \ll 1 \ . \tag{5.4}$$

Due to the fact that we expect Υ to be large and $\hat{G}_0$ to be small, this is a reasonable limit during inflation, when the typical values of the inflaton are $\phi \sim 1-10$ so the term ϕ^4 easily dominates. Therefore, we expand the square root of the potential (4.61) in terms of the quotient (5.4) up to second order, and what is found is

$$\frac{V(\phi)}{V_0} \simeq \frac{\hat{G}\phi^2}{1 + \frac{1}{2}\left(\Upsilon-1\right)\hat{G}\phi^2} \ . \tag{5.5}$$

This potential shows clearly the two possible shapes that can appear. If the product $(\Upsilon-1)\hat{G}\phi^2$ is small enough then the potential is nearly quadratic

$$V(\phi) \simeq V_0\hat{G}\phi^2 \ , \tag{5.6}$$

while if it dominates in the denominator the potential exhibits a plateau with a constant value

$$V(\phi) \simeq \frac{2V_0}{\Upsilon-1} \ . \tag{5.7}$$

Since we need $\Upsilon \sim 10^2 - 10^3$, the parameter that controls the shape of the potential is $\hat{G}_0$. When it is of the order $\hat{G}_0 \sim 10^{-6} - 10^{-5}$, the mass of the inflaton is well below M_{KK} and M_s. The DBI+CS action contains corrections to all orders in α', but only the leading-order terms contribute in this case. The result is a quadratic potential and

$\hat{G}_0$	$V_0(\Upsilon = 10^2)$	$V_0(\Upsilon = 2 \times 10^3)$
10^{-3}	4.5×10^{-8}	2.1×10^{-7}
10^{-4}	2.2×10^{-7}	6.0×10^{-7}
10^{-5}	1.9×10^{-6}	2.5×10^{-6}
10^{-6}	1.8×10^{-5}	1.9×10^{-5}

Table 1: Potential amplitudes that lead to the experimental amplitude of scalar perturbations at $N_e = 60$ e-folds.

$\hat{G}_0$	Υ	V_0	M_s	M_{KK}	$m_{\mathrm{Im}(\Phi')}$	$H(\phi_*)$	m_ϕ
10^{-5}	10^2	1.85×10^{-6}	0.25	0.23	6.58×10^{-3}	3.46×10^{-5}	6.08×10^{-6}
5×10^{-5}	10^3	6.84×10^{-7}	0.19	0.18	1.62×10^{-3}	1.68×10^{-5}	8.27×10^{-6}
10^{-3}	2×10^3	2.05×10^{-7}	0.14	0.13	8.49×10^{-4}	7.79×10^{-6}	2.02×10^{-5}

Table 2: Hierarchy of energies (in Planck units) for a quadratic potential (first row), a plateau potential (third row) and an intermediate potential (second row). The potential amplitude is normalized with the experimental scalar amplitude A_s.

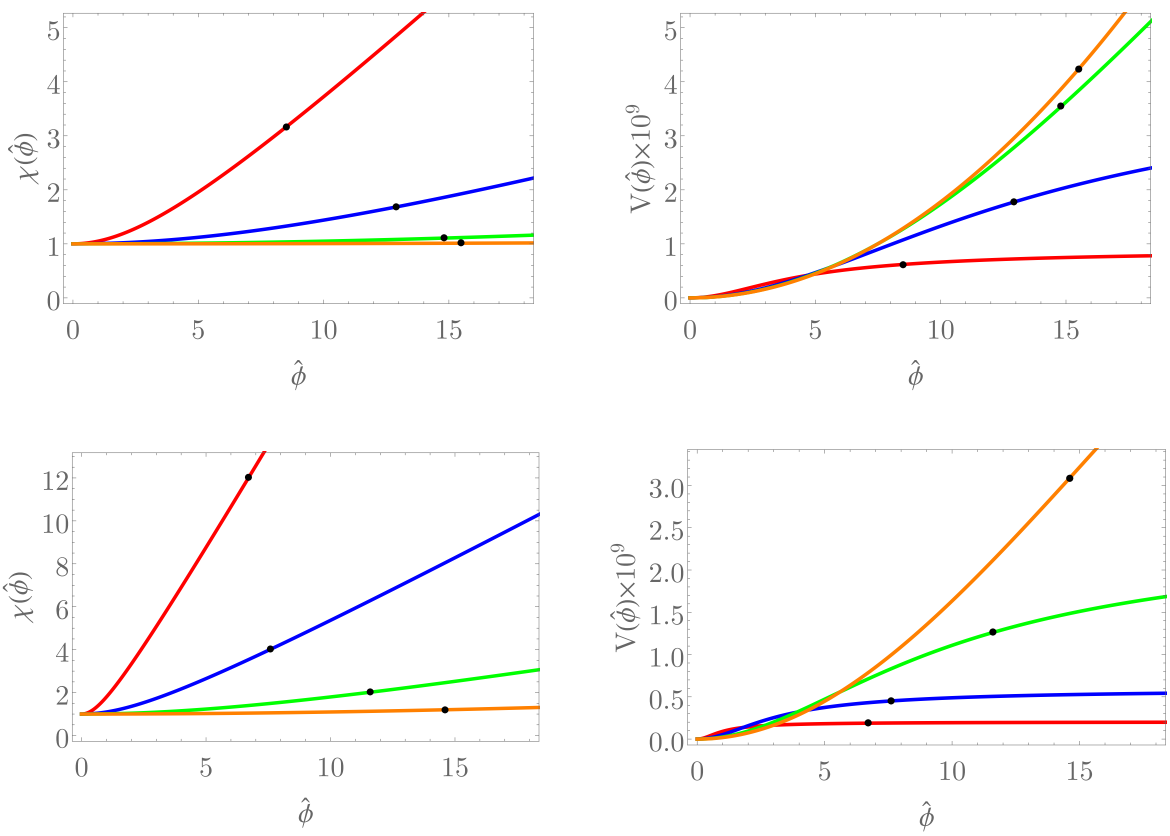

Figure 2: Kinetic and potential terms for $\Upsilon = 10^2$ (first row) and $\Upsilon = 2 \times 10^3$ (second row), and for $\hat{G}_0 = 10^{-3}$ (red), 10^{-4} (blue), 10^{-5} (green) and 10^{-6} (orange), versus the canonically normalized inflaton. The potential amplitude is normalized following table 1. The dots indicate the starting point of the last 60 e-folds of inflation.

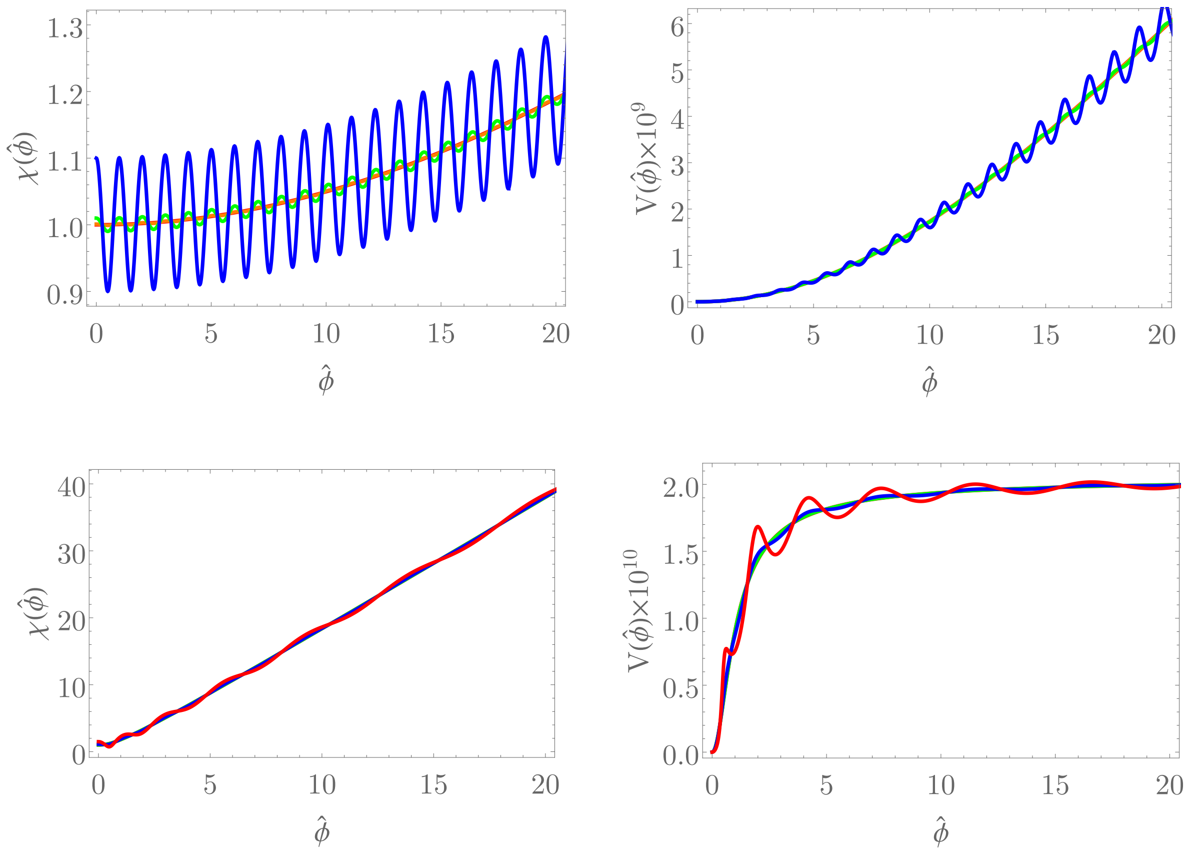

Figure 3: Kinetic and potential terms for $\hat{G}_0 = 10^{-5}, \Upsilon = 10^2$ (first row) and $\hat{G}_0 = 10^{-3}, \Upsilon = 2 \times 10^3$ (second row), and for $k = 10^{-3}$ (orange), 10^{-2} (green), 10^{-1} (blue) and 5×10^{-1} (red), versus the canonically normalized inflaton. The potential amplitude is normalized following table 1.

also a kinetic term that is almost 1. When $\hat{G}_0 \sim 10^{-4} - 10^{-3}$, for small values of ϕ the potential is still quadratic. However, the mass of the inflaton is larger and at some point the higher-order α' terms in the D7-brane action become important. Their contribution results in a change of the quadratic shape of the potential to a plateau. In addition, a quadratic behavior appears in the kinetic term instead of a constant one. Figure 2 shows the numerical results for the kinetic and potential terms taking different values of Υ and $\hat{G}_0$, where we can see directly these characteristics. They are given as functions of the canonically normalized inflaton field $\hat{\phi}$ defined in (2.15). Although the kinetic term $\chi(\hat{\phi})$ is not useful in the single-field case, here we include its plots for completeness. The model studied can be generalized to multi-field inflation, in which a canonical normalization of all the scalar fields is not possible. The kinetic terms in that case are related to the function $\chi(\phi)$, and their behavior becomes relevant. From the different parameters considered in figure 2, we choose two combinations for which the hierarchy of energies is respected, each one leading to one of the different shapes of the potential. These two choices appear in table 2 with their corresponding hierarchies. An intermediate case is also included since it will be studied in detail in section 5.4.

It is interesting to see how the non-constant warp factor modifies the quadratic and plateau potentials. In figure 3 it is shown that for a constant frequency $F = 1$ the oscillations of the warp factor are visible in $V(\hat{\phi})$ as we increase their amplitude. The quadratic potential turns out to be more sensitive to such oscillations since we can distinguish them for amplitudes of the order $k \sim 10^{-2}$. In contrast, in the plateau regime there is no such sensitivity and modulations appear for larger amplitudes of the order $k \sim 10^{-1}$. Again, the kinetic term $\chi(\hat{\phi})$ is included for completeness. Even if the oscillations are not visible at all in the potential and kinetic terms, their effects can be important in the cosmological observables, so the experimental values tightly constrain the amplitude k. Actually, considering a frequency $F = 1$ the plateau potential does not lead to inflation for $k > 10^{-1}$, and the quadratic one for $k > 2 \times 10^{-2}$. The effect of modifying the frequency is not so trivial and we explain it in detail in section 5.3.2.

5.3 Cosmological observables of the model

In this section we study the cosmological observables that emerge from the quadratic and plateau potentials that were discussed in the last section. We do not assume slow-roll conditions, so the exact numerical procedure described in 5.1 is used. The modulations that appear in the potential and kinetic terms are expected to propagate to the slow-roll parameters, and depending on their magnitude the cosmological observables are more or less affected. In figures 4, 5 there are many plots showing the results for different values of the amplitude k and the frequency F of the warp factor, and the comparison with experimental data is used to put constraints on such parameters. We consider that the inflationary model is valid if the results are in agreement with CMB observations (section 2.6) for a great part of the interval $N_e = 50 - 60$ e-folds. The reason is that there is not a theoretical or experimental motivation for a stronger fine-tuning of N_e.

5.3.1 Dependence on the amplitude

Figure 4 shows the cosmological observables obtained by fixing a frequency $F = 1$ for the warp factor and changing the amplitude of its oscillations. Focusing on the quadratic potential and in the case of a constant warp factor, the observables that are in perfect agreement with the experimental bounds are the spectral index n_s and its running. However, the tensor-to-scalar ratio r is larger than the experimental constraint $r < 0.07$ ruling out this simple case. Considering non-zero values of k leads to oscillations around the results taking a constant warp factor. Such oscillations appear strongly in the spectral index, and even more in its running, as we increase the amplitude. Firstly, it is interesting to see that an oscillatory warp factor makes it possible to move the windows of e-folds at which the results are in agreement with the observational bounds. For example, the plot of r shows that a shift effect appears at $k \sim 10^{-4}$ increasing the mean value of its oscillations. We could think of the possibility of studying higher amplitudes at which the shifting effect is a reduction of the mean value instead of an increase. The problem is that, although such regions of k could exist, the spectral index and its running impose tight constraints on the values that k can take. Amplitudes $k \gtrsim 10^{-5}$ lead to strong oscillations in the observables that put the inflationary model under pressure. We find narrow windows of N_e where the results are inside the experimental bounds, but these windows do not generally coincide for n_s and its running.

The higher sensitivity of n_s and its running to the modulations can be explained analyzing the slow-roll parameters. We have seen that the oscillatory modes of the warp factor propagate to the potential and kinetic terms. Since the time evolution of the Hubble parameter depends on these terms, we expect also oscillations to appear in H. While the first slow-roll parameter ϵ_1 feels the modulations of H and its time derivative, the second one ϵ_2 feels the ones of $\epsilon_1, \dot{\epsilon}_1$ and H. The result is an amplification of the oscillations as we move to higher order slow-roll parameters. Since r depends only on ϵ_1 at first order, the oscillations do not enter so dramatically when we increase the amplitude k, but n_s depends on ϵ_2 at first order too, and its running depends on ϵ_3 on the same footing. However, the experimental bounds on the running of n_s are relatively larger than the ones of the spectral index. Although oscillations affect strongly the running, a stronger constraint from n_s is expected.

Moving to the plateau potential, it is interesting to see that it leads to a very small tensor-to-scalar ratio for a constant warp factor. This characteristic agrees perfectly with the observational bounds in contrast to the quadratic potential. The running of n_s is also well inside the experimental bounds, but the spectral index is only valid for the window of $N_e = 50 - 56$ e-folds. Therefore, with this plateau potential the model with $k = 0$ presents a slight tension with experimental data, but we can say that it is only excluded in the region of $N_e = 56 - 60$ e-folds. The shape of $n_s(N_e)$ remains practically unaltered and only presents a vertical shift going from the quadratic potential to the plateau one. As a consequence, we expect that there is set of intermediate potentials for which n_s respect completely the cosmological bounds and r is below the experimental constraint $r < 0.07$. When the oscillatory modes are added, the cosmological observables present a different behavior compared to the one shown by the quadratic potential results. Due to the fact that the modulations are more suppressed, the full period of oscillation taking $F = 1$ does not enter in the window $N_e = 50 - 60$ e-folds. This implies the existence of ranges of amplitudes at which the effect of varying k is just a shift and a slight tilt on the cosmological observables. Choosing adequately a value of k one can introduce the observables within the experimental bounds for a particular window of e-folds. Figure 4 shows a range in which the shift due to k lowers the value of r even more, so it remains in agreement with the bound $r < 0.07$. For the spectral index n_s valid results are obtained for reduced windows of e-folds. We expect that the frequency of the oscillation modes controls this slope, so that other frequencies F can lead to a spectral index in agreement with the full window $N_e = 50 - 60$ e-folds. Finally, the shifting effect keeps the running of n_s into its observational bounds, so it does not carry any restriction. For larger amplitudes we have the same situation as with the quadratic potential. Abrupt changes of the curve of n_s and its running appear and the inflationary model is no longer valid.

To sum up, the quadratic potential presents a large tensor-to-scalar ratio that excludes the model. Even in the case of a non-constant warp factor, its value cannot be lowered since the spectral index and its running limit the possible amplitudes of its oscillations. In contrast, the plateau potential exhibits a very small value of r but n_s is not completely inside the window $N_e = 50 - 60$ e-folds for a constant warp factor. In that case the spectral index becomes the only observable that restricts the modulations of the warp factor.

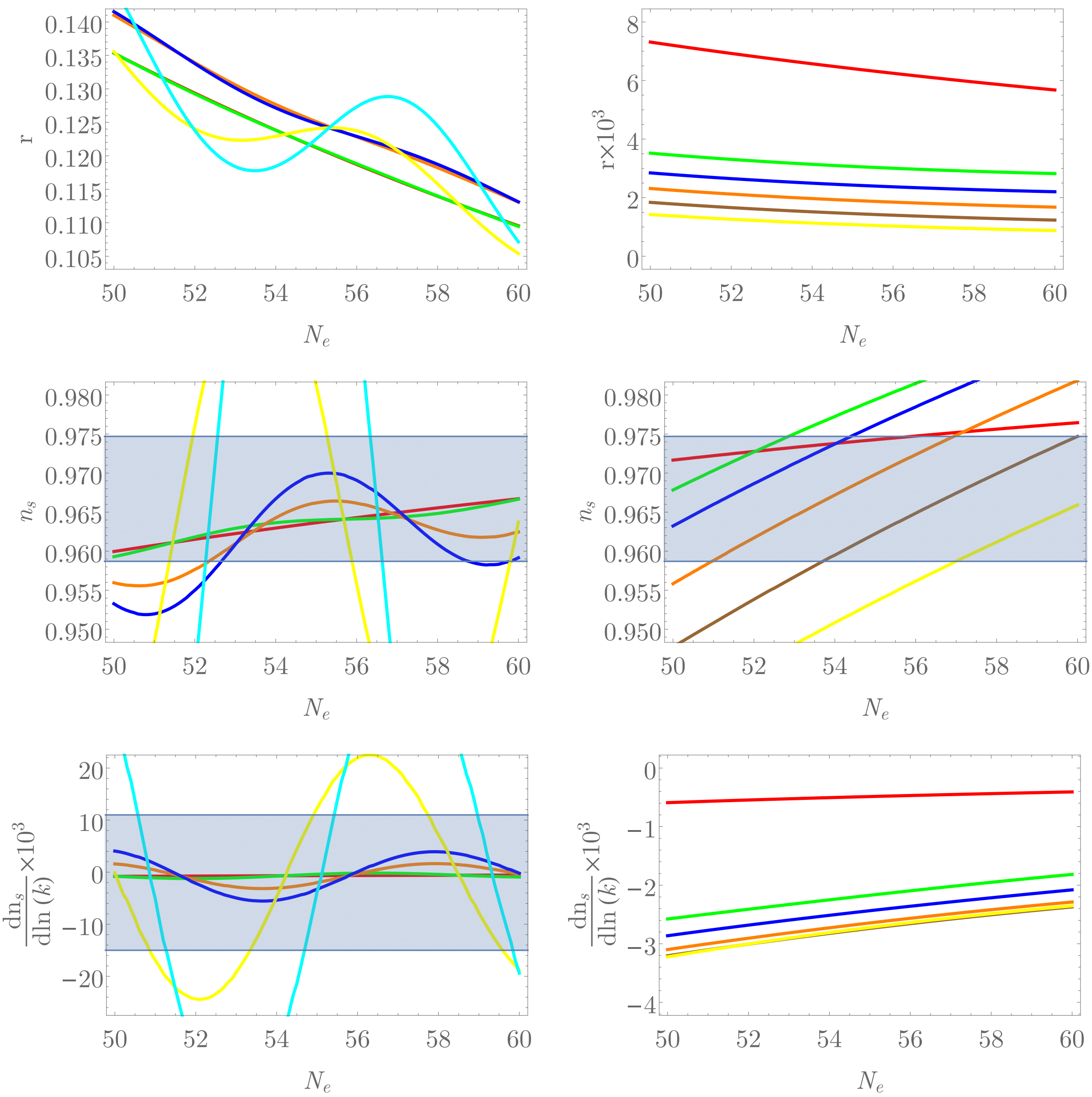

Figure 4: Cosmological observables for the quadratic (left column) and the plateau (right column) potentials considering a warp factor with a frequency $F = 1$. For the quadratic potential: $k = 0$ (red), 10^{-5} (green), 5×10^{-5} (orange), 10^{-5} (blue), 5×10^{-4} (yellow) and 10^{-3} (cyan). For the plateau potential: $k = 0$ (red), 4×10^{-2} (green), 5×10^{-2} (blue), 6×10^{-2} (orange), 7×10^{-2} (brown) and 8×10^{-2} (yellow). The blue areas indicate the experimental values given in section 2.6.

5.3.2 Dependence on the frequency

Figure 5 shows the cosmological observables obtained by fixing the amplitude k instead of the frequency of the warp factor oscillations. The results taking $k = 0$ are exactly the same as in the last subsection for both potentials, so we focus on the effect of considering a non-constant warp factor.

Firstly, an amplitude $k = 10^{-5}$ has been considered for the quadratic potential. Although it is a very small value compared to 1, we can appreciate a noticeable effect on n_s and its running. The parameter r presents small oscillations and also another slight shift for $F = 2$. Although the spectral index and its running are in good agreement till $F = 3$, the inflationary model is excluded due to the large tensor-to-scalar ratio. Futhermore, the increase of the magnitude of the oscillations as we move to higher frequencies is something expected. In the computation of the slow-roll parameters, time derivatives introduce powers of the frequency F into the amplitude of the oscillations. High frequencies lead again to abrupt oscillations that restrict the validity of the model. As we discussed before, the model is under pressure in such cases. Therefore, from cosmology we obtain a tight constraint in the oscillatory modes of the warp factor. Only the low-frequency oscillations can have a significant amplitude while the modes with larger frequencies must be highly suppressed.

Secondly, the plateau potential has been studied with a larger range of frequencies and an amplitude $k = 10^{-4}$ since it is more rigid to the warp factor oscillations. The tensor-to-scalar ratio does not impose any constraint since the oscillatory modes keep it well below the experimental bound. As for n_s and its running, only the mode with $F = 10$ lead to a consistent model for $N_e = 50 - 55$. Due to the fact that the plateu potential is more unalterable than the quadratic one, we conclude that the low-frequency modes must be less suppressed, but the high-frequency ones present the same cosmological restrictions as in the quadratic case.

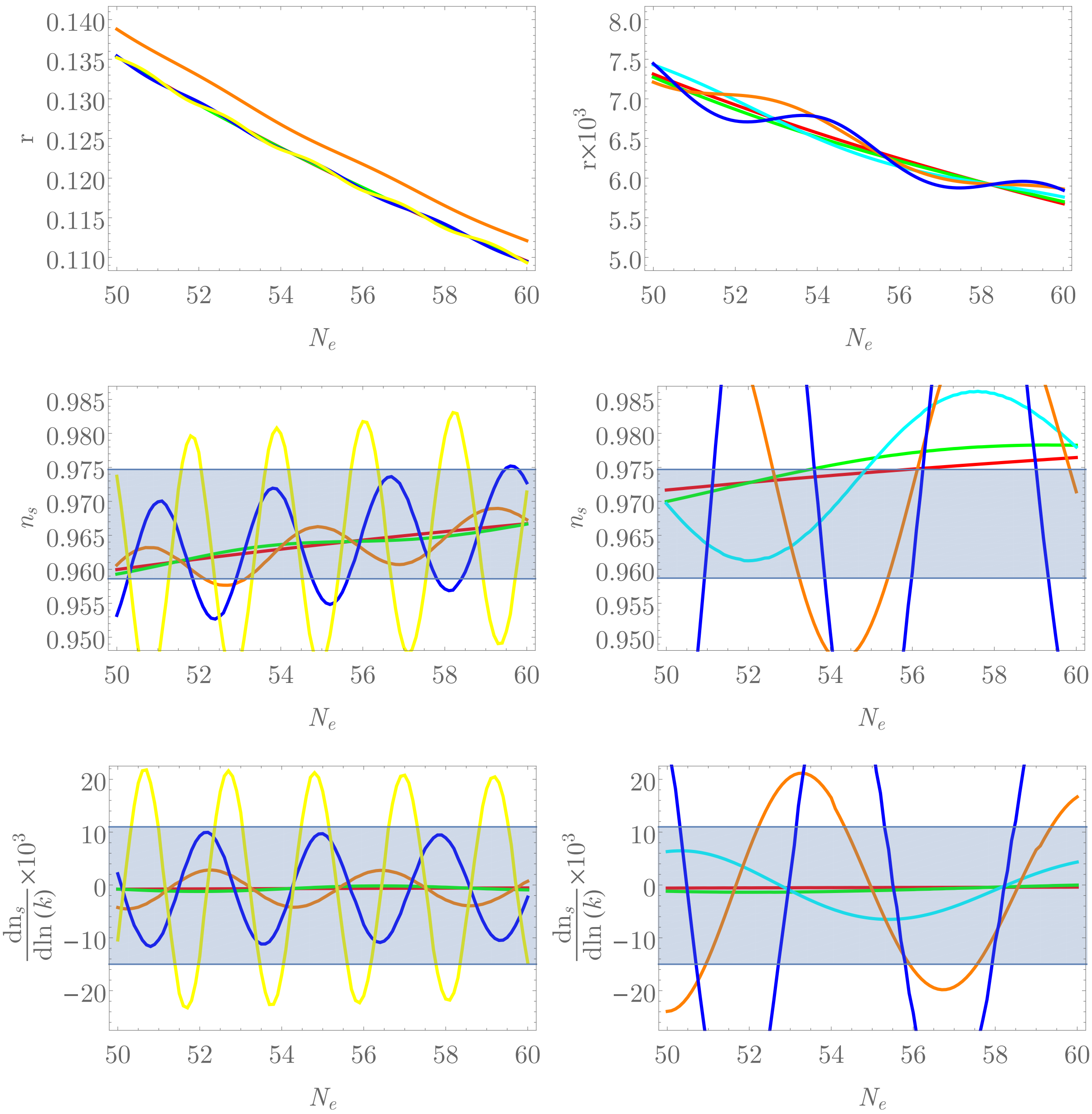

Figure 5: Cosmological observables for the quadratic (left column) and the plateau (right column) potentials considering a warp factor with amplitudes $k = 10^{-5}, 10^{-4}$, respectively. For the quadratic potential: $k = 0$ (red), $F = 1$ (green), 2 (orange), 3 (blue) and 4 (yellow). For the plateau potential: $k = 0$ (red), $F = 5$ (green), 10 (cyan), 15 (orange) and 20 (blue). The blue areas indicate the experimental values given in section 2.6.

5.4 Consistent inflationary models

From the last section we conclude that there are two possibilities to obtain an inflationary model that is consistent with CMB observations. The first one consists in generating an intermediate potential between the quadratic and plateau regimes by a different choice of $\hat{G}_0$ and Υ. In this way one can achieve a suitable model in which the resulting observables are in perfect agreement with the cosmological bounds when a constant warp factor is considered. The second one consists in introducing adequate oscillatory modes of the warp factor once the flux-dependent parameters are have been chosen. These modes can validate the inflationary models that are not consistent with a constant warp factor.

An example of an intermediate potential is given by the combination of parameters $(\hat{G}_0, \Upsilon) = (5 \times 10^{-5}, 10^3)$. The value of V_0 needed to obtain the observable A_s and the hierarchy of energies for this model can be found in table 2. It is an example in which, considering a constant warp factor, the spectral index n_s is well inside the cosmological bounds and the potential is flattened enough to generate a low tensor-to-scalar ratio. As for the second possibility, we have seen that the oscillatory modes of the warp factor cannot correct the tensor-to-scalar ratio, so we are forced to consider the plateau regime. Using the plateau potential of the last section, a modulation with $k = 2.5 \times 10^{-3}$ and $F = 2$ introduces the spectral index n_s well inside the cosmological bounds for the full window $N_e = 50 - 60$ e-folds. In figure 6 the cosmological observables that both models produce are presented showing these characteristics. A comparison with the results obtained assuming the slow-roll limit is also included. We can see that there is only a small difference when the warp factor is constant, and that this difference become smaller as we move to $N_e = 60$ e-folds. The reason is that the slow-roll limit is more satisfied as we move away from the ending point of inflation. In the case of a non-constant warp factor we expect that the slow-roll conditions are not satisfied, and we can observe that the cosmological observables obtained considering this limit present an important difference compared to the ones obtained with the exact procedure. This difference is worse as higher-order Hubble slow-roll parameters are involved in the computation of the observables.

Finally, in figure 6 there is an extra graphic included for the intermediate potential. It shows the maximum amplitude that an oscillatory mode of the warp factor can have depending on its frequency. Since the tensor-to-scalar ratio is not significantly affected by these oscillations and the running of the spectral index is not strongly constrained by cosmological observations, the observable n_s is the one that imposes these upper bounds. The criterium we have used to obtain these maximum values is the rejection of the inflationary model when its spectral index is not completely inside the cosmological bounds for the full window of $N_e = 50 - 60$ e-folds. The result confirms the qualitative analysis of the last section, since we get a maximum amplitude $k \sim 4 \times 10^{-4}$ for the first oscillatory mode, and for frequencies higher than $F = 5$ the amplitude is limited to be smaller than $k = 10^{-5}$. We find a very tight constraint on the high-frequency modes and also a significant restriction on the low-frequency ones.

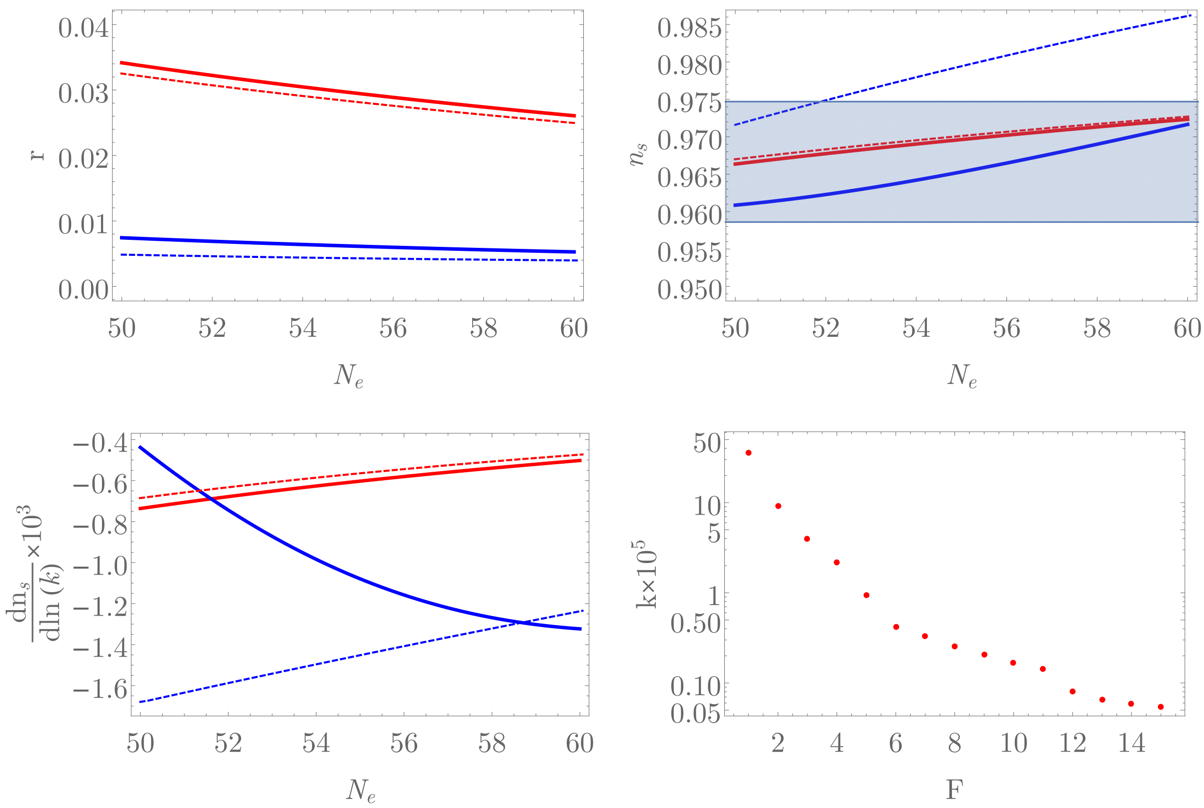

Figure 6: Cosmological observables for the intermediate potential (red) and the plateau potential (blue) and upper bound on the amplitude k of the frequency modes of the warp factor for the intermediate potential. The dashed lines show the results assuming the slow-roll limit.

6 Conclusions

Following the work done in [10], [11], we have generalized their inflationary model in which the inflaton is identified with the position of a D7-brane. In particular, we have considered a 10-dimensional background geometry with a periodic warp factor. After dimensionally reducing to four dimensions, we have computed exact solutions to the equations of motion of the inflaton field, since the slow-roll conditions are not necessarily satisfied in all cases. The model is parametrized by the flux-dependent parameters $\hat{G}_0$, Υ and by the amplitude k and the frequency F of the warp factor oscillatory modes. The parameters $(\hat{G}_0, \Upsilon)$ control the shape of the inflationary potential, which can present a quadratic or a plateau behavior. We have studied three combinations of these parameters that respect the energy hierarchy needed to use the supergravity limit and to ignore any other scalar field coming from string theory. The cosmological bounds given by CMB observations show that the plateau effect is needed to obtain a consistent tensor-to-scalar ratio, and imply a tight suppression on the maximum amplitude k of the oscillatory modes, which is stronger as their frequency increases.

Many aspects of this thesis merit further analysis. Firstly, we have assumed that a periodic warp factor is a reasonable possibility and just studied its consequences, but it can be motivated more formally from the string theory viewpoint. We expect the amplitude and the frequency of the oscillatory modes to have a physical meaning related to the position of the D-brane local sources in the compact space, since they are the ones that generate non-vanishing fluxes and lead to a non-constant warp factor. Secondly, we have also assumed all the ingredients needed to ignore all the moduli except the scalar fields related to the D7-brane position. It would be interesting to see if something important changes considering full moduli stabilization. So this would require a description in supersymmetric field theory, specifically one would have to study the correct Kähler potential and superpotential includying non-perturbative contributions. Finally, there are some remaining shortcomings to be considered, like the backreaction of the motion of the D7-brane on the surrounding geometry and the effect of higher-dimensional Planck-suppressed operators coming from the full string theory.

This work also presents the oportunity for many possible extensions. We have focused on a flux background that generates an effective single-field inflation because it provides a large mass term for one of the two scalar fields of the model. However, there is no reason to limit ourselves to this simple situation. Other flux choices that lead to more intricate multi-field inflation could also be studied considering a non-constant warp factor. In addition, future experiments like COrE, PIXIE and BICEP Array are expected to measure the tensor perturbations of inflation that are imprinted in the CMB. This can possibly test the consistency relation between the tensor-to-scalar ratio and the tensor spectral index, only valid for single-field inflation, and can give additional constraints on the parameters of the inflationary model considered.

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