Data Driven Flavour Model and its scalar potential

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Abstract

This work explores a Beyond the Standard Model realisation in which the Yukawa couplings arise as the background values of some new fields, generically dubbed flavons. The general scalar potential for these flavons is constructed abiding by the Minimal Flavour Violation hypothesis. The analysis of its minima reveals whether the mass hierarchies and mixings observed in the quark sector can be given a dynamical origin within this setup, and to which degree of naturalness. Several scenarios are reviewed, differing in the flavour symmetry group to be considered and the particular set of flavons to be added to the specific realisation. Special consideration is given to the Data Driven Flavour Model, a bottom-up approach to the flavour sector, whose scalar potential is derived and minimised in this work. Remarkably, the results of its analysis show it is possible to dynamically generate the flavour structure characterising the hadronic sector. The fine-tunings required are stronger than those present in the description of the flavour sector in the Standard Model. Nevertheless, this result constitutes a substantial improvement over generic realisations of MFV.

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1

Introduction

Modern theoretical physics is built upon the pillars of general relativity and the Standard Model (SM). Both highly successful theories capable of describing reality up to an extraordinary degree of precision, but neither exempt of its own share of problems. Gravitation has yet to overcome the challenges of quantisation, whereas from a SM point of view, problems can be labelled as either experimental or "naturalness" issues.

The first correspond to experimental results (with high significance) which are in disagreement with the predictions coming from the SM Lagrangian. Notably, neutrinos have been shown to be massive through oscillation experiments [1], leaving open the search for leptonic charge conjugation-parity (CP) violation, family transitions in the charged lepton sector and the fundamental question of whether their nature is that of Dirac or Majorana fermions. Furthermore, the visible Universe seems to be composed by much larger amounts of matter than antimatter, but how exactly did this asymmetry came to be in the first place? While the SM provides a source for this so called baryon asymmetry, it is not enough to explain the ratio observed today. On top of this, an increasing amount of cosmological and astrophysical evidence points to large quantities of non-baryonic matter, inferred only by its gravitational effects, the so called dark matter. Many viable candidates have been suggested, but we have yet to find conclusive evidence regarding its nature.

Naturalness problems are subtler and perhaps more theoretically and even aesthetically oriented issues. They correspond to phenomena whose explanation within the SM relies on certain parameters set to very specific values. The precision to which these parameters must be fixed makes them appear "unnatural" or fine-tuned, and leads us to believe that we are ultimately missing something, a deeper explanation in the form of new physics. One such complication is the so called Higgs hierarchy problem. We know the SM cannot be a complete theory (it is missing gravity after all), and thus, we can regard it as an effective theory valid up to a yet undetermined scale (not necessarily the Planck scale). Due to the scalar nature of the Higgs particle, its mass, not protected by any symmetry, picks radiative corrections of the order of the new physics scale. Fine-tuning on the bare mass in the Lagrangian is then needed to bring the Higgs mass down to its known value ($m_h \sim 125\,GeV$ [1]), how severe depending on how far the new scale is. The commonly referred to as strong CP problem is another instance of a naturalness issue. No CP violation has ever been observed in the QCD sector, even though its Lagrangian naturally includes a CP violating term. In order for the SM to comply with this observation, this term must be fine tuned out of our experimental reach. Yet another naturalness problem is that of the dark energy. The expansion of our Universe is accelerating, which, according to our current understanding of cosmology, requires the presence of a vacuum energy, whose negative pressure would be the responsible force behind this acceleration. Naive estimations of this vacuum energy within the SM lay as far as 120 orders of magnitude from the value required to fit the observed rate of expansion, evidencing our lack of understanding of the physics at play.

To complete this brief (and non-exhaustive) summary of the open challenges in the SM, we shall introduce the flavour problem, which will be the main focus of this work. The flavour sector of the SM has long been regarded as problematic. On the one hand, we face a naturalness issue commonly referred to as the flavour puzzle: there is no explanation for the heterogeneity of fermion masses and mixings, which in the SM, is merely displayed parametrically. On the other, attempts at enriching the flavour sector of the SM are often met by phenomenologically dangerous predictions of flavour violating processes.

The data driven flavour model to be explored throughout this work is a bottom-up approach tackling both of these issues, providing a dynamical origin to the Yukawa couplings sourcing fermion masses and mixings in the SM, while simultaneously avoiding phenomenologically dangerous behaviour. This is achieved by identifying the largest flavour symmetry group arising in the limit of vanishing Yukawa couplings, excluding that of the top quark, which the data suggests to be already a natural parameter of the theory. This symmetry is assumed to be exactly realised at some energy scale, and the Yukawa terms are made invariant in the high energy theory by the insertion of some new scalar fields with flavour transformation properties, the flavons. Below this scale, the flavour symmetry is spontaneously broken by the background values of the flavons, dynamically generating the Yukawa couplings, which can be now identified with the latter. The key to the good phenomenological behaviour of the model is that the symmetry constrains the appearance of flavour violating processes through higher order operators. It is also remarkable that the most stringent lower bounds on the new flavour physics scale, provided by current experimental constraints on flavour changing neutral currents, can be as low as a few $TeVs$, rendering the model a testable scenario in the not so distant future. Nevertheless, it remains to be seen whether the vacuum of the model can possibly account for the full structure of the Yukawa couplings in the SM. To answer this question is precisely the goal of this work. Through the study of the most general scalar potential involving the flavons, we will be ultimately seeking for a dynamical explanation to the origin of quark masses and mixings.

But before delving any deeper into the technicalities of the model, let us begin by briefly reviewing the status of the flavour sector within the SM along the main features of the latter, which shall be instructive before dealing with any of its proposed extensions. In what follows, we will mostly restrain our attention to the hadronic sector of the SM, for the leptonic sector is far less well understood, and the yet undetermined nature of neutrinos adds complexity to the discussion, although we shall briefly comment on its situation within the proposed flavour models.

1.1 The Standard Model

1.1.1 Interactions and boson fields

Symmetry and the gauge principle can be singled out as the main drivers of success in building our understanding of high energy particle physics. Their implementation within the framework of quantum field theory allows for the prediction of experimentally observable magnitudes such as cross sections or decay rates, among many others. The SM embodies these principles, being, at its core, a quantum field theory based on the gauge symmetry group

$$\mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{1.1}$$

This gauge group encodes the behaviour of the strong and electroweak (EW) interactions, along with the description of the spin 1 bosons which form part of the elementary SM particle content and are said to mediate them. The strong interactions manifest between those particles which transform under the gauge symmetry group $SU(3)_C$ (C stands for colour), and are thus charged under it. On the other hand, the EW interaction $SU(2)_L \times U(1)_Y$, comprises the weak isospin symmetry group $SU(2)_L$ and the weak hypercharge group $U(1)_Y$. Given the group and the coupling constants of each subgroup, hereby defined as g_s for $SU(3)_C$, g for $SU(2)_L$ and g' for $U(1)_Y$ at a certain energy scale μ, this part of the theory is completely determined. A number of spin 1 bosons arise as a consequence, one for each of the generators belonging to each subgroup, that are said to mediate the interaction. Hence, we have eight gluons for the strong interaction, three W bosons as the mediators of the weak isospin interaction, and the B boson mediating the hypercharge interaction.

We can now write the Lagrangian density for the gauge sector of the SM

$$-\mathscr{L}_g = \frac{1}{2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + \frac{1}{2}\mathrm{Tr}\left(W^{\mu\nu}W_{\mu\nu}\right) + \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \tag{1.2}$$

where μ and ν are contracted Lorentz indexes following Einstein's summation convention, and $G_{\mu\nu}$, $W_{\mu\nu}$ and $B_{\mu\nu}$ are defined as the field strengths of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ respectively in the following way

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu \boldsymbol{G}_\nu - \partial_\nu \boldsymbol{G}_\mu + i g_s \left[\boldsymbol{G}_\mu, \boldsymbol{G}_\nu\right], \\ W_{\mu\nu} &= \partial_\mu \boldsymbol{W}_\nu - \partial_\nu \boldsymbol{W}_\mu + i g \left[\boldsymbol{W}_\mu, \boldsymbol{W}_\nu\right], \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{1.3}$$

with $\boldsymbol{G}_\mu$ and $\boldsymbol{W}_\mu$ subsequently defined as

$$\boldsymbol{G}_\mu \equiv \tfrac{\lambda_i}{2} G^i_\mu, \quad \boldsymbol{W}_\mu \equiv \tfrac{\sigma_i}{2} W^i_\mu. \tag{1.4}$$

G^i_μ denote the eight gluon vector-boson fields, W^i_μ, the three weak isospin mediators, and B_μ, the hypercharge boson, whereas λ_i, the Gell-Mann matrices, are the generators of $SU(3)$, and σ_i, the Pauli matrices, the equivalent of $SU(2)$. The above Lagrangian describes the propagation and self-interaction of these fields. We shall introduce as well for later convenience the covariant derivative, which will allow for their interaction with the rest of the particle content of the SM, defined as follows

$$D_\mu \equiv \partial_\mu + ig_s \boldsymbol{G}_\mu + ig\boldsymbol{W}_\mu + ig' Q_Y B_\mu, \tag{1.5}$$

where Q_Y will be the hypercharge of the field the covariant derivative is acting upon, while $\boldsymbol{G}_\mu$ and $\boldsymbol{W}_\mu$ will only be present if the latter belongs to the fundamental representation of the corresponding gauge subgroup.

But the description of the SM interactions relies yet on another key element. Canonical bare mass terms cannot be added directly to the Lagrangian, as they are not invariant under $\mathcal{G}$. The SM circumvents this issue through the celebrated Brout-Englert-Higgs mechanism, which requires the addition of a $SU(2)_L$ doublet spinless boson (scalar under Lorentz transformations) with two complex components, denoted H, with the following transformation properties under $\mathcal{G}$

$$H \in (1, 2, 1/2)_{\mathcal{G}}\,. \tag{1.6}$$

Its implementation within the SM Lagrangian requires of the addition two new parameters

$$\mathscr{L}_H = (D_\mu H)^\dagger D^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 , \tag{1.7}$$

one with dimensions of energy, v, which implicitly defines the EW scale $v \simeq 246\, GeV$, and the dimensionless self-Higgs-coupling constant, $\lambda \simeq 0.13$ (both parameters are experimentally determined by the measurement of the Fermi constant and Higgs boson mass [1], $m_h^2/2 = \lambda v^2$). The second term above, the Higgs potential, is minimized for $\langle H^\dagger H \rangle = v^2/2$, consequently inducing a non-zero vacuum expectation value (vev) for the Higgs field. When expanding around this vacuum, its interaction with the rest of the fields will give rise to the mass terms needed to properly accommodate the physical particle spectrum. This interaction is in turn determined by the Higgs charges (or equivalently, transformation properties) under $\mathcal{G}$, which were shown in Eq.(1.6).

The Higgs non-zero vev induces the spontaneous symmetry breaking of three out of the four generators (often referred to as "directions") of the gauge group $SU(2)_L \times U(1)_Y$ of the EW interactions. Three out of the four degrees of freedom in the Higgs field would then ordinarily resolve as Goldstone bosons under the Goldstone theorem. However, since they are coupled to the EW gauge fields (through the covariant derivatives of the above Lagrangian) they end mixing with an specific combination of the W and B bosons, the

W^+, W^- and Z bosons we are familiar with, which are made massive by their inclusion, so that only the single remaining degree of freedom becomes a new scalar particle, the Higgs boson.

The combination of generators of $SU(2)_L \times U(1)_Y$ that still preserves the vacuum, and thus, defines the gauge group that remains unbroken after EW symmetry breaking, is the one corresponding to the electromagnetic charge group, $U(1)_{em}$. The combination of gauge fields pointing in this direction stays massless, and amounts to the physical photon, γ.

All in all, after the rotation from the weak interaction eigenstates to the mass eigenstates, we are left with the following physical gauge boson fields

$$\begin{aligned} \begin{pmatrix} W^+_\mu \\ W^-_\mu \end{pmatrix} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & +i \end{pmatrix} \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix}, \\ \begin{pmatrix} \gamma_\mu \\ Z_\mu \end{pmatrix} &= \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}, \end{aligned} \tag{1.8}$$

where it has been introduced the weak mixing angle, θ_W, which can be expressed in terms of the EW coupling constants g and g'.

1.1.2 Fermion fields

Next in order, completing the sequence of intrinsic angular momentum between the spin 1 vector and spin 0 scalar bosons, come the spin 1/2 fermions. They can be classified according to their transformation properties under $\mathcal{G}$, which in turn determine the way they interact with the rest of the fields of the SM. We can draw the first distinction on whether they are able or not to interact strongly under the gauge group $SU(3)_C$. Those who can, are referred to as quarks, those who cannot, we call leptons. We can split them even further according to the way they behave under the weak isospin group $SU(2)_L$. Among the quarks, we can distinguish between the doublets, Q_L, and the singlets, U_R, and D_R; whereas leptons divide into doublets, ℓ_L, and singlets, E_R. Lastly, each of these is charged differently under the hypercharge $U(1)_Y$ group. The transformation properties of the fermions in the SM are summarized in **Table 1.1**.

The representations of the non-abelian groups ($SU(3)_C$ and $SU(2)_L$) form a discrete set, e.g. the fundamental representation, the adjoint representation, etc. Every fermion in the SM either transforms in the simplest non-trivial of them, the fundamental, denoted in the above table as N for $SU(N)$,

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1/6
U_R	3	1	2/3
D_R	3	1	-1/3
ℓ_L	1	2	-1/2
E_R	1	1	-1

Table 1.1: Fermion transformation properties under $\mathcal{G}$.

or does not transform at all, acting as a singlet, denoted by 1 in the first two columns (we already used this notation for the Higgs field on Eq.(1.6)). As for the abelian gauge group $U(1)_Y$, the seemingly random choice of representations (charges) for the different fermions comes as a predictive success of the gauge principle (modulo a normalization factor). The cancellation of anomalies, or equivalently, the conservation of the symmetry at a quantum loop level, imposes hard constraints on the assignment of hypercharges. The subscripts L and R have been used to denote the left and right handed chirality components of fermions.

The interaction between fermions and gauge fields is implemented in the Lagriangian through the covariant derivative in the kinetic terms

$$\mathscr{L}_{kin} = i \sum_{\Psi} \overline{\Psi} \not{D} \Psi, \tag{1.9}$$

where $\not{D} \equiv \gamma^\mu D_\mu$, γ^μ being the Dirac matrices, and the sum over Ψ contains all the fermion fields.

But there is yet another layer to the fermion structure of the SM, namely, the flavour. Three copies of the fermion fields described in **Table 1.1** are observed in nature, with the exact same charges but disorderly masses. They are commonly known as families or generations, and can be arranged in the following way

$$\begin{array}{rl} Q_L^i = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right) & U_R^i = (u_R, c_R, t_R) \\ & D_R^i = (d_R, s_R, b_R) \;, \\ \ell_L^i = \left(\begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_L^\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix} \right) & E_R^i = (e_R, \mu_R, \tau_R) \end{array} \tag{1.10}$$

where we have used i as the index running in flavour space, denoting the up, down, charm, strange, top and bottom quarks as u, d, c, s, t and b

respectively; and in the lepton sector, the electron, the muon, the tau and their corresponding neutrinos as e, μ, τ and ν; finally displaying the whole fermion spectrum of the SM.

In the simplest realization of the SM, neutrinos are massless, and the mass terms for the rest of the fermions arise through Yukawa's interaction between these and the Higgs field, once the latter takes on the vev $\langle H\rangle \equiv \left(0, v/\sqrt{2}\right)^T$. The piece of the SM Lagrangian containing this mass generating terms reads as follows

$$-\mathscr{L}_{Yuk} = \overline{Q}_L \tilde{H} Y_U U_R + \overline{Q}_L H Y_D D_R + \overline{\ell}_L H Y_E E_R + h.c.\,, \tag{1.11}$$

where $\tilde{H} \equiv i\sigma_2 H^*$, with the second Pauli matrix, σ_2, acting on the weak isospin space; and Y_U, Y_D and Y_E denote 3×3 matrices acting on the flavour space, which we will refer to as Yukawa matrices. The Yukawa matrices encode the flavour structure of the SM, from masses to mixings, and will be one of the main objects of our study throughout this work.

1.1.3 Flavour symmetry

Given the success and predictive features granted by symmetries in their description of the SM, it is only natural to try and extend them to the flavour sector. The hadronic part of the SM Lagrangian exhibits, in the limit of vanishing quark Yukawa couplings, a flavour symmetry given by the group [2, 3]

$$\mathcal{G}_f = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}, \tag{1.12}$$

plus three extra $U(1)$ factors corresponding to the baryon number, the already reviewed hypercharge and the Peccei-Quinn symmetry [4], which are nonessential to the present discussion. On the contrary, the non-abelian subgroup $\mathcal{G}_f$ controls the flavour structure of the Yukawa matrices. The transformation properties of the quark fields under this group are shown below

$$Q_L \in (3,1,1)_{\mathcal{G}_f}\,, \quad U_R \in (1,3,1)_{\mathcal{G}_f}\,, \quad D_R \in (1,1,3)_{\mathcal{G}_f}\,. \tag{1.13}$$

A $SU(3)$ factor arises for each field in different representations of the gauge group, as showcased by the adopted notation. This is easily understood, in the absence of the Yukawa interactions, the Lagrangian in Eq.(1.9) is invariant when Ψ is transformed under unitary 3×3 matrices acting on flavour space, resulting in a $U(3)$ factor for each of the fields in the sum. The whole group then decomposes to $\mathcal{G}_f$ and the three $U(1)$ factors previously discussed.

The Yukawa terms however do not vanish, and it is in their breaking of the flavour symmetry that structure is generated between the three families. Without loss of generality, the Yukawa matrices appearing on Eq.(1.11) can be written as the product of a unitary matrix, a diagonal matrix of eigenvalues, and a different unitary matrix on the right end; explicitly

$$Y_U = \mathcal{U}_L^U y_U \mathcal{U}_R^U, \quad Y_D = \mathcal{U}_L^D y_D \mathcal{U}_R^D, \tag{1.14}$$

where $\mathcal{U}_{L.R}^{U,D}$ are the unitary matrices and $y_{U,D}$ the diagonal matrices composed by the eigenvalues of the original Yukawa matrices. The following redefinition of the quark fields in flavour space

$$Q_L \to \mathcal{U}_L^U Q_L, \quad U_R \to \mathcal{U}_R^{U\dagger} U_R, \quad D_R \to \mathcal{U}_R^{D\dagger} D_R, \tag{1.15}$$

simplifies, while leaving the rest of the Lagrangian invariant, the Yukawa matrices in Eq.(1.11) to the form

$$Y_U = y_U, \quad Y_D = \mathcal{U}_L^{U\dagger} \mathcal{U}_L^D y_D. \tag{1.16}$$

These look familiar once we switch to the most commonly used notation

$$V_{CKM} \equiv \mathcal{U}_L^{U\dagger} \mathcal{U}_L^D, \quad \begin{cases} y_U \equiv Diag\left(y_u, y_c, y_t\right), \\ y_D \equiv Diag\left(y_d, y_s, y_b\right), \end{cases} \tag{1.17}$$

where V_{CKM}, the unitary Cabibbo-Kobayashi-Maskawa quark mixing matrix, describes the mismatch between the weak interaction eigenstates and the mass eigenstates, those which propagate freely. It encodes three angles and one CP-odd phase. But let us show first the connection between the Yukawa eigenvalues and quark masses.

After the Higgs field takes on the vev $\langle H \rangle = \left(0, v/\sqrt{2}\right)^T$ and EW symmetry breaking ensues, the independent rotation of the lower component of the quark isospin doublet: $D_L \to V_{CKM} D_L$, brings us to the mass basis, rendering the Yukawa terms diagonal in flavour space

$$-\mathscr{L}_{Yuk}^q = y_i \frac{v+h}{\sqrt{2}} \overline{U}_L^i U_R^i + y_j \frac{v+h}{\sqrt{2}} \overline{D}_L^j D_R^j + h.c.\,, \tag{1.18}$$

where h is the physical Higgs boson. Quark masses are then read straightforwardly as

$$m_q = y_q \frac{v}{\sqrt{2}} \simeq y_q \, 174 GeV. \tag{1.19}$$

The values of the Yukawa couplings which reproduce the experimentally observed masses [1] turn out to be

$$\begin{aligned} (y_u, y_c, y_t) &\simeq \left(1.2 \times 10^{-5}, 7.3 \times 10^{-3}, 0.99\right), \\ (y_d, y_s, y_b) &\simeq \left(2.7 \times 10^{-5}, 5.3 \times 10^{-4}, 2.4 \times 10^{-2}\right). \end{aligned} \tag{1.20}$$

This rotation to the mass basis goes unnoticed in all but one of the rest of the terms in the Lagrangian, the one coupling the two components of the quark doublet through the EW interaction, which now reads

$$\mathscr{L}_{CC} = i \frac{g}{\sqrt{2}} \overline{U}_L V_{CKM} \not{W}^+ D_L + h.c. \tag{1.21}$$

The end result is that the flavour violating source has been effectively shifted from the mass terms to the coupling of the quarks to the $W^{\pm}$ gauge bosons. It is important to note why this mixing matrix emerges in the SM. We cannot diagonalise both Yukawa matrices at the same time because the two terms involving the up and down quarks contain the same weak isospin doublet Q_L, causing the appearance of an irreducible mixing matrix. Moreover, this mixing matrix only has an effect in the first place because U_L and D_L interact weakly, since its unitarity makes it drop form the rest of the terms in the Lagrangian. Both mass terms and $SU(2)_L$ interactions are needed in conjunction for flavour violation phenomena to manifest in the SM. The role of the unitary matrix entering the Yukawa couplings has been made apparent now, the mixing matrix parametrises the change of basis from the interaction to the mass basis.

The CKM matrix is close to the identity, with deviations given at first order by the λ parameter in the Wolfenstein parametrisation ($\lambda = \sin\theta_{12}$, where θ_{12} is the mixing angle between the first two families in the commonly used standard parametrisation)

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right). \tag{1.22}$$

The four parameters of the Wolfenstein parametrisation (adopted here because it will result convenient for the later sections) are experimentally measured to be [1]

$$\lambda \simeq 0.225, \quad A \simeq 0.8, \quad \rho \simeq 0.15, \quad \eta \simeq 0.35. \tag{1.23}$$

The CP violating phase is then determined by the combination $\rho - i\eta$.

We are finally in position to come back and review the flavour problem in the SM. Following 't Hooft's naturalness criteria [5, 6], any dimensionless parameter in the theory is expected to be generally of order one, and all dimensionful ones should be of the order of the scale(s) of the theory. Stronger than $\mathcal{O}(10\%)$ adjustments (typical Clebsch-Gordan values in any theory) are usually considered to be fine-tuned, and will be regarded undesirable. While the free parameters in the gauge sector, g_s, g, g' and λ, are smaller but of $\mathcal{O}(1)$ at the typical scale of the theory (namely, the EW symmetry breaking scale $v \simeq 246\,GeV$), the Yukawa parameters of the quark sector span, as displayed by Eq.1.20, over five orders of magnitude, and can generally be regarded as fine-tuned parameters within the theory. Moreover, while the down quark of the first generation happens to be heavier than its up counterpart (a fact which is crucial for the stability of the proton), the opposite holds true for the other two families. On the other hand, the mixing pattern of the CKM matrix stands as a mystery of its own, with no known explanation for its particular shape. For comparison, in the gauge sector, hypercharges turn out to be heavily constrained by the consistency of the theory. Furthermore, gauge invariance forces particles into representations of the group, so that the dimension of the representation dictates the number of particles, e.g. there are up and down quarks in each generation to fit the fundamental representation of $SU(2)_L$. It is this degree of fine-tuning and apparent arbitrariness characterising the parameters in the flavour sector, what causes us to refer to their enigmatic origin within the theory as the flavour puzzle.

New theories seeking for a deeper explanation to the flavour sector of the SM must also abide by vast amounts of experimental evidence, which the SM is in excellent agreement with. One such piece of evidence is the smallness of the rates of flavour changing neutral currents (FCNC), whose explanation within the SM owes to the well known Glashow-Iliopoulos-Maiani (GIM) mechanism [7]. FCNC processes occur in the SM at the loop level and end constrained by unitarity relations to be proportional to mass differences and mixing parameters, which results in a significant suppression of the predicted rates. That this behaviour is not spoiled by any extension of the SM is of paramount importance, and places, as we shall see, stringent constraints on any new model of flavour.

2

Flavour Beyond the Standard Model

The main idea behind this work is the use of symmetry in the quest for an explanation to the flavour puzzle. Many attempts have gone in this direction inspired by its success in the other sectors of the SM. The first can be dated to the late seventies, when Froggatt and Nielsen proposed the addition to the SM of a global abelian $U(1)$ factor [8]. The flavon, a new scalar field behaving as a singlet of the gauge group but charged under this symmetry, is also added to the Yukawa terms, whose invariance is then achieved by giving charges to the fermions of the SM. The Yukawa terms, rendered non-renormalisable operators by the addition of the flavon, include now a cut-off scale, Λ_f, representing the mass scale of the underlying dynamics responsible for their appearance at lower energies. The flavon, endowed with a potential, develops a non-zero vev, breaking spontaneously the flavour symmetry and giving rise to the observed fermion masses and mixings (this approach is applicable to both, quark and lepton sectors). The major drawback of this model lies on its lack of predictive power, undermined by the large amount of free parameters entering the Yukawa matrices.

Attempts were also made towards discrete non-abelian symmetries [9], specially motivated by the relatively large mixing patterns observed in the lepton sector with the data from neutrino oscillation experiments. However, their most common prediction of a vanishing or extremely small reactor angle was confronted with reality when a relatively large value was discovered for the latter in 2011 [10, 11, 12].

Non-abelian continuous symmetries have, on the other hand, managed to keep their status as viable candidates. Their main advantage being a typically more predictive framework than Froggatt-Nielsen models, but also a relatively higher degree of restriction towards the number and type of representations the fields may transform under, when compared to discrete flavour models. The hypothesis of minimal flavour violation (MFV) [3] stands as a highly successful instance of this approach, being remarkably economical on its premises but also strikingly predictive. The data driven flavour model to be proposed and studied during this work abides by its main idea, which we shall introduce next.

2.1 Minimal flavour violation

The SM can be regarded as a successful effective theory valid up to a yet to be determined cut-off energy scale, Λ. The search of physics beyond the SM strives to find effects which are present in the theory for a finite value of Λ, but vanish in the limit $\Lambda \to \infty$. The predictions of the SM, when contrasted to the available experimental evidence, provide lower bounds on Λ. As we have discussed, the flavour symmetry of the SM is already broken by the Yukawa couplings. Accordingly, it does not seem very plausible to demand that the new interactions respect flavour, a symmetry not even realized in the SM, which shall be recovered as the low energy limit of the new theory. However, generic flavour violating interactions at $\Lambda \simeq TeV$ must be avoided if we wish to conserve the good phenomenological behaviour of the SM.

The proposed solution to this apparent dilemma, is to impose that the effective theory respects the MFV hypothesis [2], defined succinctly as the requirement that all flavour and CP violating interactions must be sourced by the known structure of the Yukawa couplings.

The Yukawa interactions break $\mathcal{G}_f$ as defined in Eq.(1.12). Flavour invariance can however be formally recovered, by promoting the Yukawa couplings Y_U and Y_D to dimensionless auxiliary (spurion) fields[3], transforming as

$$Y_U \in (3, \bar{3}, 1)_{\mathcal{G}_f}, \quad Y_D \in (3, 1, \bar{3})_{\mathcal{G}_f}, \tag{2.1}$$

allowing the appearance of Yukawa interactions consistent with the flavour symmetry

$$-\mathscr{L}^q_{Yuk} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + h.c. \tag{2.2}$$

Notice the above Lagrangian describes the most general coupling of the Y fields to renormalisable SM operators, couplings of Y with the kinetic terms of quarks can be eliminated by a redefinition of the latter and Yukawa interactions with more Y insertions can be reabsorbed within the existing ones via redefinition of the Y fields.

The background values of the auxiliary Y fields can be brought, through suitable redefinition of the quark fields, to the following form

$$Y_U = y_U, \quad Y_D = V_{CKM} y_D, \tag{2.3}$$

in trivial generalisation of the procedure followed for the Yukawa matrices that culminated in Eq.1.16.

An effective theory is said to satisfy the criterion of MFV if all higher-dimensional operators constructed from SM and Y fields are invariant under CP and (formally) under the flavour group $\mathcal{G}_f$[1]. This definition of MFV leads to a realistic description of the minimal effects in flavour physics almost necessarily present in any extension of the SM with non-trivial dynamics at the scale Λ.

It is important to note MFV is not a model of flavour, and it does not determine the value of the new dynamical flavour energy scale, Λ_f, at which new flavour phenomena is expected to manifest. It is nonetheless capable of predicting precise relations between different flavour transitions, to be tested once the new physics scale becomes experimentally accessible. This is owed to the fixed flavour structure the coefficients of all SM gauge invariant operators within the MFV framework are forced into, dictated by the specific insertion of Y fields required so as to make the operator invariant under $\mathcal{G}_f$.

It is this last feature that ultimately shapes the phenomenological behaviour of MFV, making it such an interesting proposal. All new non-negligible flavour-violating effects must happen through the only relevant non-diagonal structure that can be constructed out of the contraction of Y spurion fields, i.e. the one involving two top Yukawa couplings, as any other combination results in significant amounts of suppression. The analysis of the $d = 6$ effective operators containing this structure leads to the most stringent bounds on their coefficients $1/\Lambda^2$, allowing to find lower bounds on the scale of new physics Λ. Remarkably, this scale can be as low as a few $TeVs$ under the MFV hypothesis, while still adhering to the experimental constraints on FCNC,

[1] The original MFV hypothesis [3] extends to the lepton sector, including an enlarged flavour symmetry group. However, its application within the hadronic sector only requires of the reduced symmetry group defined in this work ($\mathcal{G}_f$), which shall be enough for our purposes.

leaving the door open for many exciting theoretical possibilities to become experimentally testable in the years to come. This is in stark contrast to the predictions resulting when the analysis of the SM effective operators is carried without any further assumption. When compared with experimental constraints on FCNC, the latter place lower bounds of the order $\Lambda > 100\, TeV$ on the scale of the new physics, residing thus out of any foreseeable experimental capacity in the near future.

2.2 Dynamical Yukawa couplings

The MFV scheme demonstrated the usefulness of assigning spurious transformation properties to the Yukawa couplings and imposing formal flavour conservation at the phenomenological level. In what seems to be the next natural step, we will assume the flavour symmetry to be exact at some high energy scale Λ_f, and the Yukawa couplings to arise as the vevs of fields that had real transformation properties under this symmetry. In other words, we will promote the spurions to real fields, usually called flavons, which after the spontaneous breaking of the flavour symmetry will give rise to the known structure of the Yukawa couplings in the SM. This idea is not new, and indeed, was already present in the first formulation of MFV by Chivukula and Georgi [2], where the Yukawa couplings were the result of a fermion condensate. An extensive review on this topic can be found in ref. [13], which has been extremely helpful in the writing of this work.

To avoid the Goldstone bosons that would result from the spontaneous symmetry breaking of a continuous global flavour symmetry, it has been proposed, for instance, to gauge the symmetry. This in turn tends to induce, in practical realizations, phenomenologically dangerous FCNC mediated by the new gauge bosons. Several ways to circumvent this issue have been explored in the literature [14, 15, 16, 17, 18].

A comprehensive analysis on the flavon scalar potential of MFV specifically geared towards the dynamical generation of the Yukawa couplings in the hadronic sector was carried in ref. [19], from which a lot of results will be borrowed in the writing of this section.

After the insertion of the flavons, the Yukawa terms in the Lagrangian may be regarded as effective operators of dimension larger than four, weighted down by powers of the flavour scale Λ_f, which we will refer to as Yukawa operators. This setup could arise for instance by taking Λ_f to be the mass of heavy flavour mediators, integrated out in the low energy limit of the theory to give $d > 4$ operators involving the flavons and the SM fields. Provided the vevs to be taken by the flavon fields are smaller than Λ_f, a hierarchical analysis ordered by inverse powers of this scale is a sensible approach.

It should be noted this basic framework allows for a considerable amount of freedom model-building wise. On the one hand, the flavour symmetry to be realised at the new flavour energy scale does not necessarily need to be the full $\mathcal{G}_f$ group, although we shall review this scenario first. On the other, the precise dimension d of the Yukawa operators is not determined, and will depend on the specific set of flavons chosen to give rise to the Yukawa couplings.

In what follows we will consider the full flavour symmetry group of the hadronic sector $\mathcal{G}_f$ (as defined in Eq.(1.12)) to be exactly realized at the new scale Λ_f, for being, at least a priori, the most straightforward approach. Then, the simplest case is that of $d = 5$ Yukawa operators

$$-\mathscr{L}^q_{Yuk} = \overline{Q}_L \frac{\mathcal{Y}_U}{\Lambda_f} U_R \tilde{H} + \overline{Q}_L \frac{\mathcal{Y}_D}{\Lambda_f} D_R H + h.c.\,, \tag{2.4}$$

where the scalar flavons $\mathcal{Y}_U$ and $\mathcal{Y}_D$ transform in the bi-fundamental representation of $\mathcal{G}_f$; explicitly

$$\mathcal{Y}_U \in (3, \bar{3}, 1)_{\mathcal{G}_f}, \quad \mathcal{Y}_D \in (3, 1, \bar{3})_{\mathcal{G}_f}, \tag{2.5}$$

so that the Yukawa operators are now invariant under $\mathcal{G}_f$. The Yukawa couplings are then generated as

$$Y_U = \frac{\langle \mathcal{Y}_U \rangle}{\Lambda_f}, \quad Y_D = \frac{\langle \mathcal{Y}_D \rangle}{\Lambda_f}. \tag{2.6}$$

Another possible realization to be explored below, is to consider $d = 6$ Yukawa operators involving two scalar flavons each

$$-\mathscr{L}^q_{Yuk} = \overline{Q}_L \frac{\mathbf{y}^L_U \mathbf{y}^{R\dagger}_U}{\Lambda_f^2} U_R \tilde{H} + \overline{Q}_L \frac{\mathbf{y}^L_D \mathbf{y}^{R\dagger}_D}{\Lambda_f^2} D_R H + h.c. \tag{2.7}$$

Interestingly, the flavons can be taken now as vectors in flavour space, belonging to the fundamental representation of $\mathcal{G}_f$, just like quarks,

$$\mathbf{y}^L_{U,D} \in (3, 1, 1)_{\mathcal{G}_f}, \quad \mathbf{y}^{R\dagger}_U \in (1, 3, 1)_{\mathcal{G}_f}, \quad \mathbf{y}^{R\dagger}_D \in (1, 1, 3)_{\mathcal{G}_f}, \tag{2.8}$$

resulting in the following relations between Yukawa couplings and vevs

$$Y_U = \frac{\left\langle \mathbf{y}_U^L \right\rangle \left\langle \mathbf{y}_U^{R\dagger} \right\rangle}{\Lambda_f^2}, \quad Y_D = \frac{\left\langle \mathbf{y}_D^L \right\rangle \left\langle \mathbf{y}_D^{R\dagger} \right\rangle}{\Lambda_f^2}. \tag{2.9}$$

Only a scalar field (or an aggregate of fields in scalar configuration) can get a vev, which should correspond to the minimum of a potential. In what follows, we will turn our attention to the determination of the most general scalar potential, compatible with the flavour symmetry $\mathcal{G}_f$, that can be built for the aforementioned flavon fields. The relevant question will be whether it is possible, if at all, to accommodate the full structure of the Yukawa couplings in the SM within the minima of the scalar potential. We will derive the latter at the renormalisable level, study its vacua and discuss the degree of naturalness of the resulting solutions. When feasible, the effects of the addition of non-renormalisable terms to the Lagrangian will be explored. A thorough analysis on this subject was carried for the lepton sector in ref. [20].

It should be noted the analysis of the flavon scalar potential to be performed below may also apply to the dynamical origin of the Yukawa couplings in the lepton sector. However, the implementation of MFV in the latter [21, 22] requires of supplementary assumptions if neutrinos are assumed to be Majorana fermions, as their masses then require the extension of the SM with a new scale, that of lepton flavour violation, on top of the already introduced flavour scale. Not only that, but the starting flavour symmetry group is no longer a straightforward extension of the one in the quark sector, as right handed neutrinos are endowed with a mass not arising from interactions, but already present in the free Hamiltonian, and are thus subject to a reduced symmetry group. It is for this nuances in its analysis that the lepton sector has been left out of the scope of this work.

2.2.1 Two-family case

We shall start the discussion on the general scalar potential by studying first the two-family case, which, despite being simpler, will allow us to introduce the conventions and ideas we will be dealing with for the rest of this work. Notice we can regard this as a not too far-fetched scenario, corresponding to the limit in which the third family has decoupled, and justified by the hierarchy between quark masses and the smallness of the CKM mixing angles associated to the third family (θ_{23}, θ_{13} in the standard parametrisation).

When considering only two generations, the flavour symmetry group $\mathcal{G}_f$ is trivially reduced to

$$\mathcal{G}_f = SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(2)_{D_R}, \tag{2.10}$$

with the quark fields transforming now as

$$Q_L \in (2,1,1)_{\mathcal{G}_f}, \quad U_R \in (1,2,1)_{\mathcal{G}_f}, \quad D_R \in (1,1,2)_{\mathcal{G}_f}. \tag{2.11}$$

The flavour invariance of the Lagrangian is achieved, within the MFV scheme, through the promotion of the Yukawa couplings into spurions transforming as

$$Y_U \in (2,\bar{2},1)_{\mathcal{G}_f}, \quad Y_D \in (2,1,\bar{2})_{\mathcal{G}_f}, \tag{2.12}$$

adopting now the following background values

$$Y_U = \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix}, \quad Y_D = V_C \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}. \tag{2.13}$$

Where the mixing matrix to be reproduced:

$$V_C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{2.14}$$

is just the usual rotation among the first two families involving the Cabbibo angle θ.

2.2.1.1 Flavons in the bi-fundamental

The simplest approach promotes each Yukawa coupling into a single flavour field, generating the effective $d = 5$ Lagrangian shown in Eq.(2.4). The introduced flavons $\mathcal{Y}_U$ and $\mathcal{Y}_D$ are singlets under the SM gauge group, so that the gauge invariance of the Yukawa terms is not spoiled, but transform non-trivially under $\mathcal{G}_f$:

$$\mathcal{Y}_U \in (2,\bar{2},1)_{\mathcal{G}_f} \to \Omega_{Q_L}\mathcal{Y}_U\Omega^\dagger_{U_R}, \quad \mathcal{Y}_D \in (2,1,\bar{2})_{\mathcal{G}_f} \to \Omega_{Q_L}\mathcal{Y}_D\Omega^\dagger_{D_R}, \tag{2.15}$$

where Ω_X has been used to denote the transformation under the $SU(2)_X$ component of $\mathcal{G}_f$. The goal is then to find the scalar potential to give rise, through its minimisation, to the vev pattern displayed by Eqs. (2.6) and (2.13), spontaneously breaking the flavour symmetry and generating quark masses and mixings. The effective field theory obtained in this way at the EW scale fulfills the MFV criteria (for the two-family case).

It should be noted the model has been built in a minimalistic fashion. Both flavons develop their vevs at the same scale, and there are two of them, the minimum number required to ensure the flavour invariance of the Yukawa interactions. One could consider natural the addition of a third flavon, transforming as $\mathcal{Y}_R \in (1, 2, \bar{2})_{\mathcal{G}_f}$ to complete the basis in Eq.(2.15). Notice however this new flavon cannot contribute to the Yukawa terms at the renormalisable level, but it does introduce new operators to the MFV effective theory, mediating FCNC processes with fully right-handed quarks, which could be phenomenologically dangerous. The possibilities of adding new scales and/or this new field do not ultimately affect the flavour structure of the Yukawa couplings, and are thus not explored any further. The addition of new replicas of the bi-fundamental representations could, a priori, be helpful as a source of new scales and possible mixings, but in the end, it only trades the flavour puzzle for the flavon puzzle.

We shall thus restrict our attention to the two flavons already introduced, $\mathcal{Y}_U$ and $\mathcal{Y}_D$. The most general scalar potential for the new theory can be split in the sum of two pieces: the first corresponding to the already present in the SM Lagrangian Higgs potential (see Eq.(1.7)), responsible for the EW symmetry breaking; while the second, shall include the new terms involving the flavons, but also their interaction with the Higgs field:

$$\mathcal{V} = -\lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 + \sum_{i=4}^{\infty} \mathcal{V}^{(i)} \left[H, \mathcal{Y}_{U,D} \right]. \tag{2.16}$$

The i index has been used to denote the dimension of the scalar potential operators entering the effective field theory approach, $\mathcal{V}^{(4)}$ being an exception including all the renormalisable couplings involving H and $\mathcal{Y}_{U,D}$ of $d = 4$ and below. We will consider the flavour symmetry breaking scale to be above that of the EW symmetry breaking, for there is no reason to demand they both meet. It is however possible for the Higgs-flavon interactions to shift the values and location of the EW and flavour minima, but the flavour structure of each term is linked only to its flavon composition. After the breaking of the flavour symmetry, the terms comprised just by flavon fields in $\mathcal{V}^{(i)}$ contribute as constants to the scalar potential, whereas the ones coupling the Higgs to them can be redefined into λ or v. It is for this reason the rest of the analysis will only consider the flavon part of the scalar potential: $\mathcal{V}^{(i)}\left[\mathcal{Y}_{U,D}\right]$.

We can find the most general independent flavour invariants that enter the scalar potential by taking into account the transformation properties of the flavons shown in Eq.(2.15). At the renormalisable level, a complete set can be built out of five of them [23]

$$\begin{aligned} A_U &= \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right), & & \\ A_D &= \mathrm{Tr}\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right), & B_U &= \det\left(\mathcal{Y}_U\right), \\ & & B_D &= \det\left(\mathcal{Y}_D\right), \\ A_{UD} &= \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right), & & \end{aligned} \tag{2.17}$$

meaning any other invariant operator can be constructed as a combination of the ones shown above, e.g.

$$\mathrm{Tr}\left(\mathcal{Y}_X\mathcal{Y}_X^\dagger\mathcal{Y}_X\mathcal{Y}_X^\dagger\right) = \mathrm{Tr}\left(\mathcal{Y}_X\mathcal{Y}_X^\dagger\right)^2 - 2\,\det\left(\mathcal{Y}_X\right)^2. \tag{2.18}$$

The vev to be taken by these invariants can be expressed in terms of the SM parameters we are trying to replicate, that is, the Yukawa couplings and the Cabbibo angle. To do so, we can extract the required vev configuration for the flavons from Eqs. (2.6) and (2.13)

$$\langle\mathcal{Y}_U\rangle = \Lambda_f\begin{pmatrix} y_u & 0 \\ 0 & y_c\end{pmatrix}, \quad \langle\mathcal{Y}_D\rangle = \Lambda_f V_C\begin{pmatrix} y_d & 0 \\ 0 & y_s\end{pmatrix}, \tag{2.19}$$

leading to the following results

$$\begin{aligned} \langle A_U\rangle &= \Lambda_f^2\left(y_u^2+y_c^2\right), \quad \langle B_U\rangle = \Lambda_f^2 y_u y_c, \\ \langle A_D\rangle &= \Lambda_f^2\left(y_d^2+y_s^2\right), \quad \langle B_D\rangle = \Lambda_f^2 y_d y_s, \\ \langle A_{UD}\rangle &= \Lambda_f^4/2\left[\left(y_c^2-y_u^2\right)\left(y_s^2-y_d^2\right)\cos 2\theta + \left(y_c^2+y_u^2\right)\left(y_s^2+y_d^2\right)\right]. \end{aligned} \tag{2.20}$$

It is important to note the mixing angle only appears in the vev of A_{UD}, which was to be expected, as it is the only invariant to mix the up and down flavon sectors. Using the whole set of invariants allows for the building of the most general scalar scalar potential compatible with the flavour symmetry

$$\mathcal{V}^{(4)} = \sum_{I=U,D} \left(-\mu_I^2 A_I - \tilde{\mu}_I^2 B_I + \lambda_I A_I^2 + \tilde{\lambda}_I B_I^2\right) + g_{UD} A_U A_D + \tilde{g}_{UD} B_U B_D + \sum_{I,J=U,D} g'_{IJ} A_I B_J + \lambda_{UD} A_{UD}, \quad (2.21)$$

Where the dimensionless couplings λ, $\tilde{\lambda}$, g, g', $\tilde{g}$ would be of order one and the dimensionful parameters μ, $\tilde{\mu}$ smaller or equal than Λ_f (but around the same order of magnitude) were we imposing strict naturalness criteria. Notice the $d = 2$ terms above have been introduced with negative sign, whereas the dimensionless parameters can be, a priori, positive or negative. The ultimate goal being to construct a mexican hat-like potential for each component of the fields we wish to fix at a certain value. Consistency requires the potential to be bounded from below, hence imposing further constraints on the sign of the quartic terms (we know at least some of them will have to be positive).

We will note from the beginning, that a strict implementation of the naturalness criteria could lead at best to a strong hierarchy between the quarks, with some of them massless, and the others with masses around the same scale. Establishing a hierarchy between the relative sizes of some of the μ parameters in the potential will be needed if the observed granularity of quark mass splittings is to be reproduced, at least when limiting the analysis to the renormalisable level. The fine-tunings required (if any) to accommodate this hierarchy in the potential, will serve to gauge whether the situation has improved (or not) with respect to the flavour puzzle.

Moving on with the analysis of the potential, the relations in Eq.(2.20) allow to determine the position of the minima in terms of the physical observables, we will comment here on the most relevant physical results. Dealing first with the angular part of the potential, we can derive $\mathcal{V}^{(4)}$ with respect to the Cabibbo angle θ

$$\left.\frac{\partial \mathcal{V}^{(4)}}{\partial \theta}\right|_{min} = \lambda_{UD} \frac{\partial \langle A_{UD} \rangle}{\partial \theta} \propto \lambda_{UD} \sin 2\theta \left(y_c^2 - y_u^2\right) \left(y_s^2 - y_d^2\right) = 0. \quad (2.22)$$

It can be seen the existence of the minimum requires at least one of the following conditions to be satisfied: i) $\lambda_{UD} = 0$, ii) $\sin\theta = 0$, iii) $\cos\theta = 0$ or iv) two Yukawas in the same sector are degenerate. Condition i) removes the dependence on the angle from the potential, leaving θ undetermined. It is also not natural, in the sense that no symmetry protects this term from reappearing at the quantum level. Condition ii), on the other hand, can be interpreted as a first order solution, since the Cabibbo angle is indeed small. Higher order operators in the Λ_f expansion could potentially provide the corrections needed for a better fit to its real value, possibility that we explore below. The last two conditions lie further from reality, and higher order corrections would have to take on the task of significantly reducing the Cabibbo angle or splitting the Yukawa degeneracy respectively, rendering them unappealing. The conclusion is somewhat disappointing, given the mass splittings observed phenomenologically, the scalar potential for bi-fundamental flavons does not allow for mixing between the first two generations at the renormalisable level.

The derivatives of the potential with respect to $y_{u,c,d,s}$ shall also vanish at the minima, and provide four additional independent conditions on the parameters. Without going into explicit detail, a large, unnatural hierarchy must be imposed between the terms of the potential if a non vanishing mixing angle and distinctive Yukawas are to be obtained, otherwise a generically degenerate mass spectrum results.

The sum of these observations is that, with a natural choice for the parameters in the renormalisable scalar potential $\mathcal{V}^{(4)}$, minimisation leads to a vanishing or undetermined mixing angle, accompanied by a degenerate spectrum.

We wish to know whether the situation is improved by the addition of non-renormalisable operators to the scalar potential. Higher order traces and determinants involving the flavons can be expressed in terms of the five independent invariants already introduced in Eq.(2.17), through relations similar to that shown in Eq.(2.18); meaning the new terms will be just a composite of them. It is then easy to see the lowest higher dimensional contributions to the scalar potential have dimension six. At this order, the only terms including the mixing angle are

$$\mathcal{V}^{(6)} \supset \frac{1}{\Lambda_f^2} \sum_{I=U,D} \left(g_{UDI} A_{UD} A_I + g'_{UDI} A_{UD} B_I \right). \tag{2.23}$$

It is however clear they will share the same dependence on the Cabibbo angle previously found in Eq.(2.22) (the one coming from A_{UD}), and can be consequently absorbed into a redefinition of the low order parameter λ_{UD}. The trend for no mixing extends this way to the non-renormalisable level.

To find new angular structure we will need a term involving more than one copy of the A_{UD} invariant, the first of which appears at dimension eight:

$$\mathcal{V}^{(8)} \supset \frac{1}{\Lambda_f^4} \lambda_{UDUD} A_{UD}^2, \tag{2.24}$$

replacing Eq.(2.22) by

$$\left.\frac{\partial \mathcal{V}}{\partial \theta}\right|_{min} \propto \sin 2\theta \left(y_c^2 - y_u^2\right) \left(y_s^2 - y_d^2\right) \left(\lambda_{UD} - 2 y_c^2 y_s^2 \lambda_{UDUD} \sin^2\theta + ...\right) = 0, \tag{2.25}$$

where further suppressed terms have been neglected, and adding thus another possible solution

$$sin^2\theta \simeq \frac{\lambda_{UD}}{2 y_c^2 y_s^2 \lambda_{UDUD}}. \tag{2.26}$$

A sizable value for $\sin\theta$ within this solution would however require, when taking into account the experimental values of the Yukawa terms y_c and y_s , a completely unnatural hierarchy between the dimensionless coefficients of the $d = 4$ and $d = 8$ terms, which would have to be fine-tuned to $\lambda_{UD}/\lambda_{UDUD} \sim 10^{-10}$.

The remaining four equations, corresponding to the derivatives of the potential with respect to $y_{u,c,d,s}$, show no improvement neither. The Yukawa couplings always result from a general combination of the parameters in the scalar potential, meaning the hierarchy between them must be fine-tuned into the latter.

We can summarize these results by stating that the inclusion of higher order operators can potentially account for a non-vanishing mixing angle, coming however at the expense of naturalness. Severe fine-tunings must be enforced within the scalar potential in order to accommodate the parameters characterizing the flavour puzzle. We are thus forced to conclude, that the addition of scalar flavons transforming in the bi-fundamental representation does not lead to a satisfactory explanation for the dynamical origin of flavour.

To offer a quantifiable example, we close this section by explicitly showing a fine-tuned scalar potential allowing for hierarchical Yukawa couplings and a non-vanishing mixing angle. Two new dimension eight invariants need to be introduced:

$$A_{UDUD} = \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right), \quad A_{UUDD} = \mathrm{Tr}\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right), \tag{2.27}$$

so that the new potential reads

$$\mathcal{V} = \sum_{I=U,D}\left(-\mu_I^2 A_I + \lambda_I A_I^2 + \tilde{\lambda}_I B_I^2\right) + \frac{\lambda_{UDUD}}{\Lambda_f^4}\left(A_{UDUD} - 2A_{UUDD}\right) \\ - \varepsilon_U \tilde{\mu}_U^2 B_U - \varepsilon_D \tilde{\mu}_D^2 B_D + \varepsilon_\theta \lambda_{UD} A_{UD}, \tag{2.28}$$

where $\varepsilon_{U,D,\theta}$ parametrise the required fine-tunings. Omitting any analytical detail, the minimisation of the above potential leads to the following Yukawa eigenvalues and Cabibbo angle

$$\begin{aligned} y_u &\simeq \frac{\varepsilon_U}{\sqrt{2}}\frac{\sqrt{\lambda_U}\tilde{\mu}_U^2}{\tilde{\lambda}_U\mu_U\Lambda_f}, \quad y_d \simeq \frac{\varepsilon_D}{\sqrt{2}}\frac{\sqrt{\lambda_D}\tilde{\mu}_D^2}{\tilde{\lambda}_D\mu_D\Lambda_f}, \\ y_c &\simeq \frac{\mu_U}{\sqrt{2\lambda_U}\Lambda_f}, \quad y_s \simeq \frac{\mu_D}{\sqrt{2\lambda_D}\Lambda_f}, \\ \sin^2\theta &\simeq \varepsilon_\theta \frac{\lambda_{UD}}{\lambda_{UDUD}y_c^2 y_s^2}. \end{aligned} \tag{2.29}$$

When the suppression parameters take on the values $\varepsilon_U \sim 10^{-3}$, $\varepsilon_D \sim 5\times 10^{-2}$ and $\varepsilon_\theta \sim 10^{-10}$; and the following fractions are suitably chosen as $\mu/\left(\sqrt{\lambda}\Lambda_f\right) \sim \tilde{\mu}/\left(\sqrt{\tilde{\lambda}}\Lambda_f\right) \sim 10^{-3}$, the right Cabibbo angle and hierarchy among the quark masses can be recovered.

Notice this example has been exclusively shown for illustrative purposes, the potential in Eq.(2.28) considers two dimension 8 operators, while simultaneously ignoring many other consistent with the flavour symmetry $\mathcal{G}_f$, whose appearance should be weighted in the first place against similarly ordered quantum corrections. Even further fine-tunings were ultimately required to fully recreate the flavour structure of the two-family case, including highly unnatural $\mathcal{O}\left(10^{-10}\right)$ values, which evidences the lack of attractiveness of the present solution.

2.2.1.2 Flavons in the fundamental

Seeking for an alternative set of flavons to more faithfully reproduce the structure of the Yukawa couplings, we consider next the possibility that they arise from an aggregate of fields, specifically two of them, thus increasing the dimension of the Yukawa operators to $d = 6$. The simplest case involves two scalar flavons transforming in the fundamental representation of $\mathcal{G}_f$ as a replacement for the Yukawa couplings. The particular realization to be explored will be the one already shown in Eqs. (2.7) and (2.9).

Notice a minimalistic approach would just require the existence of three flavons, one for each component of the flavour symmetry, transforming as $(2,1,1)$, $(1,2,1)$ and $(1,1,2)$ under $\mathcal{G}_f$. Such a setup leads however to no mixing between the first two families at the renormalisable level, as will be shown below. It is for this reason, that an additional field in the $(2,1,1)$ representation will be considered, so that the up and down sector Yukawa terms do no longer have to share the same flavon, explicitly

$$\mathbf{y}^L_{U,D} \in (2,1,1)_{\mathcal{G}_f}, \quad \mathbf{y}^R_U \in (1,2,1)_{\mathcal{G}_f}, \quad \mathbf{y}^R_D \in (1,1,2)_{\mathcal{G}_f}. \tag{2.30}$$

These fields are vectors under the flavour symmetry. The only physical invariants that can be constructed out of a set of vectors are their norms and the relative angles between them. Any matrix built out of the multiplication of two vectors has only one non-vanishing eigenvalue, a result that is independent on the number of the dimensions of the space considered. Out of this fact alone we can already conclude that the Yukawa couplings resulting from a construction such as the one shown in Eq.(2.7), only provide mass to one quark at a time. Since there are two Yukawa couplings, one associated to the "up" sector and the other, associated to the "down" sector, two massive quarks result, an up- and a down-type quark, while the others will display vanishing masses. This is an encouraging first step, as hierarchy arises naturally within this model between quarks with the same electric charge. It also means the 2×2 Yukawa matrices generated after the breaking of the flavour symmetry contain many unphysical parameters, unphysical in the sense that they can be redefined away by a suitable choice of fields, as we show next. Without losing generality, we can parametrise the vevs to be taken by the flavons as

$$\left\langle \mathbf{y}^X_I \right\rangle \equiv \left|\mathbf{y}^X_I\right| V^X_I \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.31}$$

where X and I stand for L, R and U, D respectively, V^X_I are 2×2 unitary matrices, and $\left|\mathbf{y}^X_I\right| \equiv \left|\left\langle \mathbf{y}^X_I \right\rangle\right|$ is consequently defined as the norm of the vev to be taken by the flavon. After the breaking of the flavour symmetry, it is possible to redefine the quark fields as

$$Q_L \to V_U^L Q_L, \quad U_R \to V_U^R U_R, \quad D_R \to V_D^R D_R, \tag{2.32}$$

bringing Eq.(2.7), back to the usual appearance of the Yukawa Lagrangian in the SM

$$-\mathscr{L}_{Yuk} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + h.c.\,, \tag{2.33}$$

only that now, the Yukawa matrices are given by

$$Y_U = \frac{\left|\mathbf{y}_U^L\right|\left|\mathbf{y}_U^R\right|}{\Lambda_f^2}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, \quad Y_D = \frac{\left|\mathbf{y}_D^L\right|\left|\mathbf{y}_D^R\right|}{\Lambda_f^2} V_U^{L\dagger} V_D^L \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}. \tag{2.34}$$

It should be noted again that different flavons could potentially be assigned different scales. Instead, we will assume for simplicity that the scales of the four flavons are relatively close, regarding Λ_f as a first order approximation to all of them. Eq.(2.34) illustrates explicitly the two points made above. A hierarchy naturally arises between the two families here considered, without imposing any constraint on the parameters of the scalar potential. On the other hand, it is now easy to see why we demanded the addition of a second flavon to transform in the $(2, 1, 1)$ representation. The product $V_L^{U\dagger} V_L^D$ is a non-trivial unitary matrix characterizing the structure of the mixing between the two generations, i.e. the Cabibbo angle (an additional complex phase can be easily absorbed into the fields in the two-family case under consideration). But the only reason it appeared in the first place, is because we could not redefine away the unitary matrix of two flavons at the same time with that of the single left field Q_L. Had we just considered one flavon that would not have been the case. This leaves us with a very clear geometrical interpretation: the misalignment between the vevs of the $\mathbf{y}^L$ flavons in $SU(2)_{Q_L}$ space is ultimately responsible for the appearance of mixing between the two families, parametrised by the Cabibbo angle. The mixing matrix for the two-family case in Eq.(2.14), to appear in the weak interaction terms (see Eq.(1.21)), can be then identified as

$$V_C = V_U^{L\dagger} V_D^L. \tag{2.35}$$

The general scalar potential once the new flavons are introduced will be analogous to that already shown for the bi-fundamental in Eq.(2.16)

$$\mathcal{V} = -\lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 + \sum_{i=4}^{\infty} \mathcal{V}^{(i)} \left[H, \mathbf{y}_{U,D}^{L,R} \right].,\tag{2.36}$$

and just as before terms mixing Higgs and flavons will not be explicitly discussed, as previous considerations regarding the scale separation between EW and flavour symmetry breaking still hold. We will thus refer to $\mathcal{V}^{(i)} \left[\mathbf{y}_{U,D}^{L,R} \right]$ going forward, as the piece of the potential containing the terms exclusively involving flavons.

As previously argued, the list of flavour invariants that can be built out of the set of fundamental flavons is restricted to their norms and the relative angle between the two sharing the same vector space, i.e. $\mathbf{y}_U^L$ and $\mathbf{y}_D^L$, explicitly

$$\mathbf{y}_U^{L\dagger} \mathbf{y}_U^L, \quad \mathbf{y}_U^{R\dagger} \mathbf{y}_U^R, \quad \mathbf{y}_D^{L\dagger} \mathbf{y}_D^L, \quad \mathbf{y}_D^{R\dagger} \mathbf{y}_D^R, \quad \mathbf{y}_U^{L\dagger} \mathbf{y}_D^L.\tag{2.37}$$

Any other flavour invariant operator can be constructed out of these five independent building blocks. On the other hand, the combination of Eqs. (2.13) and (2.34) leads to the following relations between the vevs to be taken by the flavons and the physical parameters to be reproduced

$$\left| \mathbf{y}_U^L \right| \left| \mathbf{y}_U^R \right| = \Lambda_f^2 y_c, \quad \left| \mathbf{y}_D^L \right| \left| \mathbf{y}_D^R \right| = \Lambda_f^2 y_s,\tag{2.38}$$

whereas further comparison with Eq.(2.31) allows to write

$$\left\langle \mathbf{y}_U^{L\dagger} \mathbf{y}_D^L \right\rangle = \cos\theta \left| \mathbf{y}_U^L \right| \left| \mathbf{y}_D^L \right|.\tag{2.39}$$

As can be seen, the scalar potential depends only on three out of the five physical parameters within this setup, the two Yukawa couplings corresponding to the heavier second generation quarks, given by the product of the up or down sector flavon moduli, and the Cabibbo angle. The appearance of the latter in the above equation confirms our geometrical intuition, the mixing angle is simply the angle defined in flavour space by the left up and down vectors. The relevant question is the same it was before, can the flavour invariant potential accommodate the minimum required for the flavour structure of the SM to be reproduced? and if so, to what degree of naturalness?

We thus turn our attention to the building of the scalar potential. To this end, it will prove useful to arrange the flavour invariants in Eq.(2.37) as a vector, denoting it by $\mathbf{y}^2$, and by $\langle \mathbf{y}^2 \rangle$, its vev

$$\mathbf{y}^2 \equiv \left(\mathbf{y}_U^{L\dagger}\mathbf{y}_U^L, \mathbf{y}_U^{R\dagger}\mathbf{y}_U^R, \mathbf{y}_D^{L\dagger}\mathbf{y}_D^L, \mathbf{y}_D^{R\dagger}\mathbf{y}_D^R, \mathbf{y}_U^{L\dagger}\mathbf{y}_D^L\right)^{\mathrm{T}}, \tag{2.40}$$

$$\left\langle \mathbf{y}^2 \right\rangle \equiv \left(\left|\mathbf{y}_U^L\right|^2, \left|\mathbf{y}_U^R\right|^2, \left|\mathbf{y}_D^L\right|^2, \left|\mathbf{y}_D^R\right|^2, \left\langle \mathbf{y}_U^{L\dagger}\mathbf{y}_D^L\right\rangle\right)^{\mathrm{T}}. \tag{2.41}$$

This condensed notation allows to write the most general scalar potential at the renormalisable level as

$$\mathcal{V}^{(4)} = -\frac{1}{2}\sum_I \left(\mu_I^2 \mathbf{y}_I^2 + h.c.\right) + \sum_{I,J} \lambda_{IJ} \left(\mathbf{y}_I^2\right)^* \mathbf{y}_J^2 = -\frac{1}{2}\left(\mu^2 \mathbf{y}^2 + h.c.\right) + \left(\mathbf{y}^2\right)^\dagger \lambda \mathbf{y}^2, \tag{2.42}$$

where the indices I, J run over the five entries of the above defined vectors, UL, UR, DL, DR, UD; and the parameters of the potential have been arranged into the vector μ^2 and the 5×5 hermitian matrix λ. We will additionally demand the latter to be positive definite in order to ensure the potential is bounded from below. The most general renormalisable potential is thus comprised by a total of 20 invariant operators. Being positive definite, λ is also invertible, so the addition of a constant term to the potential allows us to bring the above expression to the following form

$$\mathcal{V}^{(4)} = \left(\mathbf{y}^2 - \frac{1}{2}\lambda^{-1}\mu^2\right)^\dagger \lambda \left(\mathbf{y}^2 - \frac{1}{2}\lambda^{-1}\mu^2\right). \tag{2.43}$$

The minimum, and thus the vev configuration of the flavons, is now straightforwardly extracted as

$$\left\langle \mathbf{y}^2 \right\rangle = \frac{1}{2}\lambda^{-1}\mu^2. \tag{2.44}$$

The Yukawa couplings and the Cabibbo angle can be then expressed in terms of the parameters of the potential by comparison of the above equation to Eqs. (2.38) and (2.39), yielding

$$\begin{gathered} y_c^2 = \frac{1}{4\Lambda_f^4}\left(\lambda^{-1}\mu^2\right)_{UL}\left(\lambda^{-1}\mu^2\right)_{UR}, \quad y_s^2 = \frac{1}{4\Lambda_f^4}\left(\lambda^{-1}\mu^2\right)_{DL}\left(\lambda^{-1}\mu^2\right)_{DR}, \\ \cos\theta = \frac{(\lambda^{-1}\mu^2)_{UD}}{\sqrt{(\lambda^{-1}\mu^2)_{UL}\,(\lambda^{-1}\mu^2)_{DL}}}. \end{gathered} \tag{2.45}$$

Remarkably, $\cos\theta \sim \mathcal{O}(1)$, and a sizable mixing angle can be naturally attained even at the renormalisable level. On the other hand, Yukawa eigenvalues result $\mathcal{O}\left(\mu^2/\lambda\Lambda_f^2\right)$, and are fixed to their observed values, Eq.(1.20), by imposing

$$\begin{cases} \sqrt{(\lambda^{-1}\mu^2)_{UL}\,(\lambda^{-1}\mu^2)_{UR}} \sim 10^{-2}\Lambda_f^2 & \text{for } y_c, \\ \\ \sqrt{(\lambda^{-1}\mu^2)_{DL}\,(\lambda^{-1}\mu^2)_{DR}} \sim 10^{-4}\Lambda_f^2 & \text{for } y_s. \end{cases} \tag{2.46}$$

The situation is thus quantitatively improved with respect to the bi-fundamental approach, as the hierarchy between the first two families and non-negligible mixing are naturally explained within this model without the need of strong fine-tunings.

Notice however, that the first generation has remained massless throughout the analysis of the potential at the renormalisable level. An interesting question is whether it is possible for non-renormalisable corrections to induce small masses for the lightest quarks. These corrections could potentially manifest in two ways: in the form of new, higher order operators in the potential modifying its minima, and/or as higher order contributions to the Yukawa operators.

The first cannot possibly give masses to the first generation, as the discussion from Eqs. (2.31) to (2.34) still holds, being purely based on mathematical arguments, regardless of the vevs to be taken by the flavons. Their only effect is to modify the moduli and unitary matrices appearing in Eq.(2.31), thus redefining the mixing angle θ and second family Yukawas y_c and y_s, without changing the rank of the Yukawa matrices.

As for the second, the transformation properties of the fundamental flavons imply that higher order contribution to the Yukawa operators can only be constructed by further insertions of $\mathbf{y}^\dagger\mathbf{y}$ into the renormalisable operators. Such insertions do not however modify the flavour structure of the Yukawa matrices, but simply shift the values of the combinations appearing to the left in Eq.(2.38), merely redefining the two heavier Yukawa couplings y_c and y_s.

To summarise, non-renormalisable interactions cannot induce Yukawa couplings for the quarks of the first generation, which remain massless even after their consideration.

Lastly, we shall briefly explore the phenomenological consequences resulting from the addition of the fundamental flavons presently under discussion, to be compared with those expected from the adoption of the bi-fundamental flavons introduced in the previous section. For the latter, the list of effective operators matches that of the original MFV proposal [3], whereas the fundamental flavons allow for the construction of higher-dimension invariants yet exhibiting lower dimension than those that can be built out of the bi-fundamental. Consider for instance the following two basic bilinear FCNC structures

$$\begin{cases} \overline{D}_R \mathcal{Y}_D^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger Q_L & d=6, \\ \overline{D}_R \mathbf{y}_D^R \mathbf{y}_U^{L\dagger} Q_L & d=5, \end{cases} \tag{2.47}$$

where the mass dimension of the flavour invariant has been made explicit. Effective operators mediating FCNC processes, constructed out of the above invariants, exhibit thus different degrees of suppression. In this sense, the identification of the Yukawa couplings with an aggregate of two flavons departs from the simplest realisation of MFV, featuring potentially different phenomenology and consequently, providing a way to discriminate between fundamental and bi-fundamental origin.

2.2.2 Three-family case

The analysis of the three-family case for both bi-fundamental and fundamental representations will be, for the most part, a straightforward extension on the one performed for the first two families in the previous section, but two major differences warrant our attention.

In the two-family case, the smallness of the largest Yukawa coupling y_c, allowed to safely neglect next to leading order contributions in the Yukawa perturbative expansion, but this is no longer the case once the top Yukawa y_t of order $\mathcal{O}(1)$ is introduced. The latter forces, a priori, to consider all orders in the expansion. Thankfully, there is a way to circumvent this apparent predicament: the Cayley-Hamilton identity [24], which states a general 3×3 matrix X must satisfy the following relation

$$X^3 - \mathrm{Tr}\left(X\right) X^2 + \frac{1}{2} X \left(\mathrm{Tr}^2\left(X\right) - \mathrm{Tr}\left(X^2\right)\right) - \det\left(X\right) = 0. \tag{2.48}$$

The use of the latter allows to express any power X^n, with $n > 2$, in terms of the identity, X, and X^2, with coefficients involving the traces of X

and X^2 and the determinant of X. The flavour invariant products $\mathcal{Y}^\dagger\mathcal{Y}$ or $\mathbf{y}\mathbf{y}^\dagger$ will be the target of its application when considering bi-fundamental or fundamental flavons respectively.

The second difficulty involves the appearance of a physical complex phase in the CKM quark mixing matrix, together with three other mixing angles[2]. Whether or not this CP-violating phase can be accommodated within the potential will be explored below.

2.2.2.1 Flavons in the bi-fundamental

We consider now the original flavour symmetry group $\mathcal{G}_f$ as shown in Eq.(1.12), under which the new bi-fundamental flavons will transform as

$$\mathcal{Y}_U \in (3,\bar{3},1)_{\mathcal{G}_f} \to \Omega_{Q_L}\mathcal{Y}_U\Omega^\dagger_{U_R}, \quad \mathcal{Y}_D \in (3,1,\bar{3})_{\mathcal{G}_f} \to \Omega_{Q_L}\mathcal{Y}_D\Omega^\dagger_{D_R}, \qquad (2.49)$$

where the Ω_X matrices refer now to the triplet transformation under the $SU(3)_X$ component of the flavour group. The Yukawa Lagrangian will be that already shown in Eq.(2.4), and consequently, the same is true for the relations between the Yukawa couplings and the flavon vevs, laid out in Eq.(2.6). After the spontaneous breaking of the flavour symmetry, the vevs induced shall be those that reproduce the observed quark masses and CKM mixing matrix, embedded in the Yukawa matrices as displayed by Eqs. (1.16) and (1.17). Just as before, we could potentially complete the set of flavons by the addition of a third RH field $\mathcal{Y}_R \in (1,3,\bar{3})_{\mathcal{G}_f}$, departing from the minimal setup. This possibility is however not considered as it is not relevant to the flavour structure of the Yukawa couplings, to which it cannot contribute neither at $\mathcal{O}(1/\Lambda_f)$ nor at $\mathcal{O}(1/\Lambda_f^2)$ order, or equivalently, through $d=5$ or $d=6$ Yukawa operators respectively.

We will restrict the explicit analysis to the part of the renormalisable scalar potential constructed purely out of flavon fields, omitting the terms involving the SM Higgs, as previous arguments still hold. A complete and independent basis of flavour invariants is given now by the following seven operators

[2]This section will follow the PDG conventions [1] for the CKM matrix parametrisation, involving the three standard mixing angles θ_{12}, θ_{23}, θ_{13}, and a complex phase δ.

$$
\begin{aligned}
A_U = \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), &\quad A_D = \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right), \\
B_U = \det\left(\mathcal{Y}_U\right), &\quad B_D = \det\left(\mathcal{Y}_D\right), \\
A_{UU} = \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right), &\quad A_{DD} = \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right), \\
A_{UD} = \mathrm{Tr}&\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right),
\end{aligned}
\tag{2.50}
$$

whose vevs can be expressed in terms of the physical parameters to be reproduced through Eq.(2.6):

$$
\begin{aligned}
\langle A_U \rangle = \Lambda_f^2 \left(y_u^2 + y_c^2 + y_t^2\right), &\quad \langle A_D \rangle = \Lambda_f^2 \left(y_d^2 + y_s^2 + y_b^2\right), \\
\langle B_U \rangle = \Lambda_f^3 y_u y_c y_t, &\quad \langle B_D \rangle = \Lambda_f^3 y_d y_s y_b, \\
\langle A_{UU} \rangle = \Lambda_f^4 \left(y_u^4 + y_c^4 + y_t^4\right), &\quad \langle A_{DD} \rangle = \Lambda_f^4 \left(y_d^4 + y_s^4 + y_b^4\right),
\end{aligned}
\tag{2.51}
$$

with all the angular dependence encoded within the invariant

$$
\begin{aligned}
\langle A_{UD} \rangle / \Lambda_f^4 = &- \sum_{i<j} \left(y_{u_i}^2 - y_{u_j}^2\right)\left(y_{d_i}^2 - y_{d_j}^2\right) \sin^2\theta_{ij} + \\
&+ \sum_{i<j,k} \left(y_{d_i}^2 - y_{d_k}^2\right)\left(y_{u_j}^2 - y_{u_k}^2\right) \sin^2\theta_{ik} \sin^2\theta_{jk} + \\
&- \left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right) \sin^2\theta_{12} \sin^2\theta_{13} \sin^2\theta_{23} + \\
&+ \frac{1}{2}\left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right) \cos\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin\theta_{13},
\end{aligned}
\tag{2.52}
$$

where $i, j, k = 1, 2, 3$. The first term above generalises that of the two-family case in Eq.(2.20), grouping all the terms with single angular dependence.

The most general scalar potential at the renormalisable level is built out of the new invariants

$$
\mathcal{V}^{(4)} = \sum_{I=U,D} \left(-\mu_I^2 A_I + \tilde{\mu}_I B_I + \lambda_I A_I^2 + \lambda_I' A_{II}\right) + g_{UD} A_U A_D + \lambda_{UD} A_{UD}.
\tag{2.53}
$$

It is worth noting that the $B_{U,D}$ invariants have mass dimension three now, instead of two, as they had for the two-generation case, forbidding the appearance of B_X^2 or $B_X A_Y$ terms at the renormalisable level.

The minimisation of the potential is involved and will not be explicitly shown, but we shall comment some features characterising its solutions, which are relatively similar to that of the two-family case. The equations resulting from the derivatives of the potential with respect to the mixing angles favor no mixing, i.e. $\sin\theta_{12} = \sin\theta_{23} = \sin\theta_{13} = 0$, and while there are solutions with non-vanishing angle configurations due to the last term of Eq.(2.52), constituting a novel possibility not present in the two-family case, they are not representative of reality, involving large mixing angles for the third family. The situation is not improved neither for the Yukawa eigenvalues, which tend to be degenerate in most of the parameter space. There is however a region in the latter allowing for a non vanishing Yukawa coupling per up and down sector, for non strictly zero but constrained $\tilde{\mu}$. But manually imposing the hierarchy between the top and bottom masses is still required, which further demands $g_{UD} < y_b^2/y_t^2$. so that the fine-tunings enforced are in the end comparable to those of the two-family case. Furthermore, the similarities extend beyond this point, as initially vanishing $\sin\theta$ at the renormalisable level do not receive higher order corrections from the addition of non-renormalisable terms to the potential. Nevertheless, they do help in the scenario involving the fine-tuned choice of the parameters g_{UD} and $\tilde{\mu}$, by enabling the introduction of lighter Yukawas, although no hierarchy among the first two generations can be naturally enforced.

The above results can be summarised by stating that the sole consideration of the two bi-fundamental scalars presently under discussion cannot possibly account for the flavour structure of masses and mixings in the SM, when giving a dynamical origin to the Yukawa couplings under naturalness criteria.

2.2.2.2 Flavons in the fundamental

We now contemplate the scenario involving flavons transforming in the fundamental representation of $\mathcal{G}_f$ for the three-family case. Non-trivial mixing requires the adoption of at least four vectors, a left and a right pair of up and down flavons

$$\mathbf{y}^L_{U,D} \in (3,1,1)_{\mathcal{G}_f}, \quad \mathbf{y}^R_U \in (1,3,1)_{\mathcal{G}_f}, \quad \mathbf{y}^R_D \in (1,1,3)_{\mathcal{G}_f}. \tag{2.54}$$

Once they develop vevs, the flavour symmetry is spontaneously broken, and the Yukawa couplings arise as in Eq.(2.9). As was previously discussed for the two-family case, it is possible, without the loss of generality, to parametrise the flavon vevs as

$$\left\langle \mathbf{y}_I^X \right\rangle \equiv \left|\mathbf{y}_I^X\right| V_I^X \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \tag{2.55}$$

where X and I stand for L, R and U, D respectively, and V_I^X are now 3×3 unitary matrices. Proceeding in analogous fashion to the way we did for the two-family case, we can redefine the quark fields to bring the Yukawa matrices to the following form

$$Y_U = \frac{\left|\mathbf{y}_U^L\right| \left|\mathbf{y}_U^R\right|}{\Lambda_f^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad Y_D = \frac{\left|\mathbf{y}_D^L\right| \left|\mathbf{y}_D^R\right|}{\Lambda_f^2} V_U^{L\dagger} V_D^L \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{2.56}$$

Remember that their origin as the product of two vectors is ultimately the reason the Yukawa matrices cannot possibly have more than one non-vanishing eigenvalue. A result independent on the parametrisation adopted for the flavon vevs, which in this case has been chosen just so this fact is openly displayed. Notice as well, that the non-vanishing eigenvalue in both the up and down sectors has been chosen to be that of the third generation, as it involves the heaviest quarks.

From the structure of the above Yukawa matrices, it follows that the flavon vevs have not completely broken the flavour symmetry, leaving a residual $SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(2)_{D_R}$ among the first two generations, which remain massless and thus indistinguishable. As a consequence, any rotation in the 12 sector has no observable physical effect. Additionally, it should be noted that only one physical mixing angle can possibly be accounted for within this setup, as it can only be sourced from the misalignment between the two vevs of the flavons sharing the $SU(3)_{Q_L}$ component of $\mathcal{G}_f$, $\mathbf{y}_U^L$ and $\mathbf{y}_D^L$. It follows that the most logical choice for the latter, is to be identified with the θ_{23} CKM mixing angle, as it is only second in size to θ_{12} (the hierarchy goes as $\theta_{12} > \theta_{23} > \theta_{13}$), so that the quark mixing matrix resulting from this scenario reads

$$V_U^{L\dagger} V_D^L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} . \tag{2.57}$$

The analysis of the scalar potential is an exact copy of that shown for the two-family case, thus reaching identical solution (see Eq.(2.45)), with the trivial replacement of y_c, y_s for y_t, y_b and θ_{23} substituting the Cabibbo angle. Both, the largest hierarchy among generations and a $\cos\theta_{23}$ of order $\mathcal{O}(1)$, are in this way naturally explained without the need of any fine-tunings, the main drawbacks being it is impossible to generate lighter fermion masses even after the inclusion of non-renormalisable terms and of course, the lack of the full mixing pattern (including the complex phase).

On the other hand, the partial breaking of the flavour symmetry group displayed by Eq.(2.56) can open interesting possibilities from a model building perspective, suggesting it could be a good idea to consider a multi-step approach, where different components of the flavour group $\mathcal{G}_f$ are broken at different scales. Such scenario would however require a significantly enlarged flavon spectrum and involve a far less generic potential, losing attractive as a possible solution.

Another sensible approach, specially from an effective Lagrangian point of view, could be to consider the combination of bi-fundamental and fundamental flavons, working at $\mathcal{O}(1/\Lambda_f^2)$, and thus simultaneously including the contribution from $d = 5$ and $d = 6$ Yukawa operators. This possibility shares however the same pitfalls of the last one, as the analysis of the potential would be likely involved, and the direct connection between the minima of the latter and the flavour structure of the Yukawa couplings is lost.

3

Data driven flavour model

The disappointing conclusion that can be extracted from the last few sections is that the simplest promotion of MFV from a low-energy description to a well-defined high-energy theory does not lead to a satisfying explanation, in the form of a dynamical origin for the Yukawa couplings, for the flavour structure of the SM. The spontaneous breaking of the full flavour symmetry group $\mathcal{G}_f$, exhibited by the latter in the limit of vanishing mass terms, is caused, in this realization, by the vevs of the simplest sets of scalar flavons consistent with the flavour symmetry, i.e. bi-fundamental and fundamental flavons. The first, when considering the minimal amount of flavons required (that is, two), lead naturally to a degenerate quark mass spectrum, with vanishing or undetermined mixing angles. Whereas the second, when considered in the amount of four, to source the appearance of a non-vanishing mixing angle, are able to provide a strong mass hierarchy between quarks of the same charge, resulting in a distinctly heavier quark in each sector, but ultimately fall short in generating the Yukawa couplings for the lighter quarks and accounting for the full mixing pattern. A departure from the at least a priori, most straightforward approaches, seems to be warranted if we wish to accommodate the full flavour spectrum within this setup.

The data driven flavour model [25] is an agnostic, bottom-up approach, built solely on the available experimental data regarding the flavour sector. Avoiding any fine-tuning on the Yukawa couplings, the quark mass hierarchy suggests that only the top Yukawa term should arise at the renormalisable level, as it involves the only $\mathcal{O}(1)$ parameter satisfying naturalness criteria (see Eq.(1.20)). The starting assumption will be that this term is already invariant under the considered flavour symmetry without any flavon insertion, which are however still needed for the rest of the quarks. The model distinguishes in this way the third family from the two lighter ones from the very beginning, naturally describing a top Yukawa of order 1, and thus avoiding any technical difficulty regarding its eventual perturbative expansion in terms of the flavour scale Λ_f.

The effective operators of the model are forced into fixed flavour structures, dictated by the specific insertion of flavon fields required so as to make the operator invariant under the new flavour symmetry. As a consequence, the analysis of the $d = 6$ effective operators reveals the most stringent lower bound on the new flavour physics scale Λ_f to be of the order of a few $TeVs$, coming from the requirement of complying with experimental FCNC constraints. In this sense, the phenomenology of the model is very similar to that of the MFV framework, providing an exciting opportunity to find new physics in the near future.

The goal of this section is to explicitly show the construction of the data driven flavour model, eventually turning to the analysis of its flavon potential. Is it possible for the latter to harbor the minimum required for the origin of flavour as we know it? And if so, to what degree of naturalness? These are the questions we would like to answer in this work.

Let us introduce first the flavour symmetry that will characterise the model. Consider the following Yukawa Lagrangian for the quark sector

$$-\mathscr{L}^q_{Yuk} = y_t \overline{Q}^3_L \tilde{H} t_R + \Delta\mathscr{L}^q_{Yuk} + h.c.\,, \tag{3.1}$$

where Q^3_L denotes the third generation left-handed $SU(2)_L$ doublet, t_R the $SU(2)_L$ singlet right-handed top quark (see Eq.(1.10)), and $\Delta\mathscr{L}^q_{Yuk}$ contains the Yukawa couplings responsible for the rest of quark masses and mixing. The new flavour symmetry $\mathcal{G}_q$ is then formally defined as the largest non-Abelian quark flavour symmetry group consistent with the whole Lagrangian in the absence of $\Delta\mathscr{L}^q_{Yuk}$, that is

$$\mathcal{G}_q = SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(3)_{D_R}. \tag{3.2}$$

The transformation properties of the quark content in the SM under this group are as follows: the fields Q^3_L and t_R are now singlets under $\mathcal{G}_q$; the left-handed quarks of the first two families, arranged as the flavour vector $Q^{1,2}_L$, transform as a doublet under $SU(2)_{Q_L}$; the right-handed quarks of the up sector, analogously bundled into $U^{1,2}_R$, transform as a doublet under $SU(2)_{U_R}$;

	$SU(2)_{Q_L}$	$SU(2)_{U_R}$	$SU(3)_{D_R}$
$Q_L^{1,2}$	2	1	1
Q_L^3	1	1	1
$U_R^{1,2}$	1	2	1
t_R	1	1	1
D_R	1	1	3
$\Delta\mathcal{Y}_U$	2	$\overline{2}$	1
$\Delta\mathcal{Y}_D$	2	1	$\overline{3}$
$\mathbf{y}_D$	1	1	$\overline{3}$

Table 3.1: Quark and flavon transformation properties under $\mathcal{G}_q$.

and lastly, the three right-handed down quarks, grouped into D_R, transform as a triplet of $SU(3)_{D_R}$.

The Yukawa terms in $\Delta\mathscr{L}^q_{Yuk}$ are rendered invariant by the insertion of three new flavons, transforming as follows under $\mathcal{G}_q$

$$\Delta\mathcal{Y}_U \in (2,\overline{2},1)_{\mathcal{G}_q}, \quad \Delta\mathcal{Y}_D \in (2,1,\overline{3})_{\mathcal{G}_q}, \quad \mathbf{y}_D \in (1,1,\overline{3})_{\mathcal{G}_q}. \tag{3.3}$$

The complete list of quark and flavon transformation properties under $\mathcal{G}_q$ has been summarised in **Table 3.1** for easier reading.

The resulting Lagrangian involves the following $d=5$ Yukawa operators

$$\Delta\mathscr{L}^q_{Yuk} = \overline{Q}_L^{1,2}\tilde{H}\frac{\Delta\mathcal{Y}_U}{\Lambda_f}U_R^{1,2} + \overline{Q}_L^{1,2}H\frac{\Delta\mathcal{Y}_D}{\Lambda_f}D_R + \overline{Q}_L^{3}H\frac{\mathbf{y}_D}{\Lambda_f}D_R. \tag{3.4}$$

Notice each Yukawa coupling arises from the vev of a single flavon. In this sense, the model has been kept as minimal as possible. The Yukawa matrices generated in this fashion, once the spontaneous breaking of the flavour symmetry takes place, can be read from the above Lagrangian to be

$$Y_U = \begin{pmatrix} \langle\Delta\mathcal{Y}_U\rangle & 0 \\ 0 & y_t \end{pmatrix}, \quad Y_D = \begin{pmatrix} \langle\Delta\mathcal{Y}_D\rangle \\ \langle\mathbf{y}_D\rangle \end{pmatrix}. \tag{3.5}$$

The vevs to be adopted by the flavons in order to reproduce the mixings and masses of the quark sector (except that, of course, of the top), can be now extracted by simple comparison with the Yukawa matrices of the SM (see Eqs. (1.16) and (1.17)):

$$\langle \Delta \mathcal{Y}_U \rangle = \Lambda_f \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix},$$
$$\langle \Delta \mathcal{Y}_D \rangle = \Lambda_f V_{CKM}^{2\times 3} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \tag{3.6}$$
$$\langle \mathbf{y}_D \rangle = \Lambda_f V_{CKM}^{1\times 3} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix},$$

where $V_{CKM}^{2\times 3}$ and $V_{CKM}^{1\times 3}$ denote the 2×3 matrix and 1×3 vector corresponding to the first two and third rows of the CKM matrix respectively.

3.1 The scalar potential

We wish to know whether the above vevs can be naturally accommodated for the flavons within the model, so we turn now our attention to the building of the most general scalar potential compatible with the flavour symmetry $\mathcal{G}_q$. At the renormalisable level, a complete and independent basis for the latter is given by the following seven invariant operators

$$\begin{gathered} A_U = \mathrm{Tr}\left(\Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger\right), \quad A_D = \mathrm{Tr}\left(\Delta\mathcal{Y}_D \Delta\mathcal{Y}_D^\dagger\right), \\ A_{UU} = \mathrm{Tr}\left(\Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger \Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger\right), \quad A_{DD} = \mathrm{Tr}\left(\Delta\mathcal{Y}_D \Delta\mathcal{Y}_D^\dagger \Delta\mathcal{Y}_D \Delta\mathcal{Y}_D^\dagger\right), \\ A_{UD} = \mathrm{Tr}\left(\Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger \Delta\mathcal{Y}_D \Delta\mathcal{Y}_D^\dagger\right), \\ B_D = \mathbf{y}_D \mathbf{y}_D^\dagger, \quad B_{DD} = \mathbf{y}_D \Delta\mathcal{Y}_D^\dagger \Delta\mathcal{Y}_D \mathbf{y}_D^\dagger, \end{gathered} \tag{3.7}$$

meaning any other flavour invariant can be constructed by combination of the above. For example, $\det(\Delta\mathcal{Y}_U)$ while also a $d < 4$ invariant, can be written in terms of A_U and A_{UU} by virtue of the following relation

$$\det(\Delta\mathcal{Y}_U) = \frac{1}{2}\left(\mathrm{Tr}\left(\Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger\right)^2 - \mathrm{Tr}\left(\Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger \Delta\mathcal{Y}_U \Delta\mathcal{Y}_U^\dagger\right)\right), \tag{3.8}$$

so that, if it were to be present in the Lagrangian, in addition to the aforementioned invariants, it could just be absorbed into a redefinition of the coefficients accompanying A_U^2 and A_{UU}. Once the flavons acquire vevs as in Eq.(3.6), these invariants can be expressed in terms of the physical observables to be reproduced:

$$
\begin{aligned}
\langle A_U\rangle =&\Lambda_f^2\left(y_u^2+y_c^2\right),\\
\langle A_D\rangle =&\Lambda_f^2\left(y_d^2+y_s^2+y_b^2-y_d^2\left|V_{31}\right|^2-y_s^2\left|V_{32}\right|^2-y_b^2\left|V_{33}\right|^2\right),\\
\langle A_{UU}\rangle =&\Lambda_f^4\left(y_u^4+y_c^4\right),\\
\langle A_{DD}\rangle =&\Lambda_f^4\Big(\left(y_d^2\left|V_{11}\right|^2+y_s^2\left|V_{12}\right|^2+y_b^2\left|V_{13}\right|^2\right)^2+\\
&+\left(y_d^2\left|V_{21}\right|^2+y_s^2\left|V_{22}\right|^2+y_b^2\left|V_{23}\right|^2\right)^2+\\
&+2\left(y_d^4\left|V_{11}\right|^2\left|V_{21}\right|^2+y_s^4\left|V_{12}\right|^2\left|V_{22}\right|^2+y_b^4\left|V_{13}\right|^2\left|V_{23}\right|^2\right)\Big),\\
\langle A_{UD}\rangle =&\Lambda_f^4\left(y_u^2\left(y_d^2\left|V_{11}\right|^2+y_s^2\left|V_{12}\right|^2+y_b^2\left|V_{13}\right|^2\right)+\right.\\
&\left.+y_c^2\left(y_d^2\left|V_{21}\right|^2+y_s^2\left|V_{22}\right|^2+y_b^2\left|V_{23}\right|^2\right)\right),\\
\langle B_D\rangle =&\Lambda_f^2\left(y_d^2\left|V_{31}\right|^2+y_s^2\left|V_{32}\right|^2+y_b^2\left|V_{33}\right|^2\right),\\
\langle B_{DD}\rangle =&\Lambda_f^4\left(y_d^4\left|V_{31}\right|^2\left(1-\left|V_{31}\right|^2\right)+\right.\\
&\left.+y_s^4\left|V_{32}\right|^2\left(1-\left|V_{32}\right|^2\right)+y_b^4\left|V_{33}\right|^2\left(1-\left|V_{33}\right|^2\right)\right),
\end{aligned}
\tag{3.9}
$$

where V_{ij} denote the entries of the CKM matrix, whose unitarity relations have been used to simplify some of the expressions above. Notice the dependence on the complex phases of the entries in the CKM matrix is lost at the level of the potential, as only their moduli appears within the invariants.

Previous considerations regarding the separation between the EW and flavour symmetry breaking energy scale and the role of the Higgs-flavon interaction terms in the scalar potential still hold. Consequently, any Higgs-involving term will be disregarded in the forthcoming analysis. Having found the basis of flavour invariants in Eq.(3.7), we are now in position to write the most general scalar potential allowed by the flavour symmetry at the renormalisable level:

$$
\begin{aligned}
\mathcal{V}^{(4)}=\sum_{I=U,D}&\left(-\mu_I^2A_I+\lambda_IA_I^2+\lambda_{II}A_{II}+g'_{ID}A_IB_D\right)-\tilde{\mu}_D^2B_D+\tilde{\lambda}_DB_D^2+\\
&+\lambda_{UD}A_{UD}+\lambda'_{DD}B_{DD}+g_{UD}A_UA_D.
\end{aligned}
\tag{3.10}
$$

As a quick reminder, strict naturalness criteria would require all dimensionless couplings – λ, g, and variations – to be $\mathcal{O}(1)$ parameters of the theory, whereas the dimensionful couplings – μ – are naturally expected to be smaller or equal than the flavour scale Λ_f, while sharing the same order of magnitude.

3.2 Minimisation of the potential

We now turn our attention to the minimisation of the scalar potential in Eq.(3.10). The use of the relations introduced in Eq.(3.9), allows to determine the position of the potential minima in terms of the physical observables. We then face a purely mathematical exercise: given $\mathcal{V}^{(4)}$ as a function of the variables to result in the five lightest Yukawa eigenvalues and the three mixing angles and complex phase describing the CKM matrix, is there any combination of the parameters in the potential – λ, g, μ, and variations, that allows for a minimum to exist at the precise point of the nine-dimensional variable space corresponding to their exact values in the SM? To answer this question is the goal of this section, and ultimately, of this work. Special consideration will be given to the degree of naturalness exhibited by the resulting solutions.

The first step is to derive $\mathcal{V}^{(4)}$ with respect to each of the physical variables, from which we obtain nine conditions that must be met by the minima of the potential. It is at this point that we are met with the first difficulty of the analysis, the potential in Eq.(3.10) is comprised by thirteen terms involving intricate dependencies on the physical variables, making impracticable what should be, a priori, a straightforward analytical procedure. Note, however, that the values of the physical parameters in the SM span several orders of magnitude (see Eqs. (1.20) and (1.22)). Hinted by this fact and inspired by the helpfulness of the Wolfenstein parametrisation, we recognize an expansion in terms of powers of a shared suppression parameter, to be a sensible approach. For convenience, this parameter is chosen to be that already appearing in Wolfenstein's parametrisation, λ (see Eq.(1.23)). The idea is to reparametrise the dependence on the physical variables, switching to a new set where the order of magnitude of the corresponding observable has been factorised in powers of λ, so that the masses and mixing pattern of the SM arise when each of these new variables take $\mathcal{O}(1)$ values. For the sake of simplicity and in order to obtain a manageable expansion in powers of λ, we consider the following parametrisation as a good first approximation to the CKM matrix in the SM (see Eq.1.22)

$$V_{CKM} \simeq \begin{pmatrix} 1 & \theta_c\lambda & \theta_b\lambda^3 \\ -\theta_c\lambda & 1 & \theta_a\lambda^2 \\ \theta_b\lambda^3 & -\theta_a\lambda^2 & 1 \end{pmatrix}, \tag{3.11}$$

where $\theta_{a,b,c}$ are the variables chosen to introduce the dependence on the physical mixing observables to the potential. Notice only three variables have been adopted to describe the four physical parameters in the CKM matrix, which could be potentially concerning, as we are implicitly imposing an additional condition to the analysis. In practice, the effect of this choice is negligible, as the V_{31} entry of the CKM matrix appears always suppressed by y_d in the potential, as can be seen from the expressions involving the vevs of the flavons in Eq.(3.9).

As for the Yukawa eigenvalues, we simply factorise their relative "sizes" in terms of integer powers of λ:

$$y_u = y'_u\lambda^8, \quad y_c = y'_c\lambda^3, \quad y_d = y'_d\lambda^7, \quad y_s = y'_s\lambda^5, \quad y_b = y'_b\lambda^3. \tag{3.12}$$

The new variables $y'_{u,c,d,s,b}$ will be $\mathcal{O}(1)$ valued when the right masses are generated for the corresponding quarks.

Once the above substitutions are performed on the flavon vevs in Eq.(3.9), the equations for the minima of the scalar potential $\mathcal{V}^{(4)}$ can be obtained by deriving the latter with respect to the newly introduced variables. The well defined hierarchy of the eight resulting equations in terms of powers of λ, will allow now to treat and solve them perturbatively. Notice we are implicitly assuming strict naturalness criteria for the parameters of the potential, i.e. λ, g, $\mu/\Lambda_f \sim \mathcal{O}(1)$, in this first approach to the analysis. The power expansion in terms of λ would otherwise crumble, as we have not extracted any power of the latter from the parameters, and we are relying on it as our gauging tool for the relative size of the terms in the potential. Staying within analytical reach, the conditions for the existence of the minima with the new parametrisation read

$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial \theta_a} = -2\frac{\mu_D^2}{\Lambda_f^2}y_b'^2\theta_a\lambda^{10} + \mathcal{O}(\lambda^{14}), \quad \frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial \theta_b} = -2\frac{\mu_D^2}{\Lambda_f^2}y_b'^2\theta_b\lambda^{12} + \mathcal{O}(\lambda^{18}),$$
$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial \theta_c} = -2\frac{\mu_D^2}{\Lambda_f^2}y_s'^2\theta_c\lambda^{12} + \mathcal{O}(\lambda^{16}),$$
$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial y_u'} = -2\frac{\mu_U^2}{\Lambda_f^2}y_u'\lambda^{16} + \mathcal{O}(\lambda^{22}), \quad \frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial y_d'} = -2\frac{\mu_D^2}{\Lambda_f^2}y_d'\lambda^{14} + \mathcal{O}(\lambda^{20}),$$
$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial y_s'} = -2\frac{\mu_D^2}{\Lambda_f^2}y_s'\lambda^{10}\left(1+\theta_c^2\lambda^2\right) + \mathcal{O}(\lambda^{16}),$$
$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial y_c'} = -2\frac{\mu_U^2}{\Lambda_f^2}y_c'\lambda^{6} + 2g_{UD}'y_b'^2y_c'\lambda^{12} + 4\left(\lambda_U+\lambda_{UU}\right)y_c'^3\lambda^{12} + \mathcal{O}(\lambda^{16}),$$
$$\frac{1}{\Lambda_f^4}\frac{\partial \mathcal{V}^{(4)}}{\partial y_b'} = -2\frac{\tilde{\mu}_D^2}{\Lambda_f^2}y_b'\lambda^{6} - 2\frac{\mu_D^2}{\Lambda_f^2}y_b'\left(\theta_a^2\lambda^{10}+\theta_b^2\lambda^{12}\right) + 2g_{UD}'y_c'^2y_b'\lambda^{12} + 4\tilde{\lambda}_D y_b'^3\lambda^{12} + \mathcal{O}(\lambda^{16}). \tag{3.13}$$

We do not need to go any further in the expansion. Under strict naturalness criteria, it is already clear that no minima can possibly exist for $\mathcal{O}(1)$ valued $y'_{u,c,d,s,b}$ and $\theta_{a,b,c}$, since no cancellation can be arranged among the strongly hierarchical terms above. This is as intuitively expected, quadratic terms dominate the potential, suppressed by fewer powers of Yukawa eigenvalues and mixing entries than the quartic ones. The dimensionful parameters in the potential, μ_U, μ_D and $\tilde{\mu}_D$, accompanying the first, will certainly need to be fine-tuned to some extent if the mixing pattern and quark masses of the SM are to be accommodated.

To find desirable solutions for the new variables, we can try selectively suppressing, with powers of λ, the parameters appearing in the troubling terms of the minimum conditions, the objective being to bring enough of them to the same order of λ, so that non-vanishing $\mathcal{O}(1)$ solutions become possible for the variables. Note, however, that the conditions for the minima often share the same parameters between them. It may very well be the case that suitable solutions cannot simultaneously be achieved for all the variables, this procedure is in no way guaranteed to succeed. Nevertheless, it shall be instructive to show one such possibility.

Consider, for instance, that based on our previous observations, we decide to impose the following hierarchy between the parameters of the potential

$$\tilde{\mu}_D = \delta\tilde{\mu}_D\lambda^3, \quad \mu_U = \delta\mu_U\lambda^3, \quad \mu_D = \delta\mu_D\lambda^2, \tag{3.14}$$

so that the $\mathcal{O}(1)$ values correspond now to the fractions $\delta\mu/\Lambda_f$. This accomplishes two things: first, it delays the appearance of structure in the equations concerning the three mixing parameters and the three lightest Yukawa eigenvalues, which could not possibly lead to an appealing solution in the lowest orders of the expansion; but also, it manages to bring the only two non-vanishing derivatives appearing now below $\mathcal{O}(\lambda^{14})$, involving the charm and bottom Yukawa couplings, to the following form

$$\begin{aligned}\frac{1}{\Lambda_f^4}\left.\frac{\partial\mathcal{V}^{(4)}}{\partial y'_c}\right|_{min} &= \left(-2\frac{\delta\mu_U^2}{\Lambda_f^2}y'_c + 2g'_{UD}y_b'^2y'_c + 4\left(\lambda_U+\lambda_{UU}\right)y_c'^3\right)\lambda^{12} + \mathcal{O}(\lambda^{16}) = 0,\\ \frac{1}{\Lambda_f^4}\left.\frac{\partial\mathcal{V}^{(4)}}{\partial y'_b}\right|_{min} &= \left(-2\frac{\delta\tilde{\mu}_D^2}{\Lambda_f^2}y'_b + 2g'_{UD}y_c'^2y'_b + 4\tilde{\lambda}_Dy_b'^3\right)\lambda^{12} + \mathcal{O}(\lambda^{14}) = 0.\end{aligned} \tag{3.15}$$

The above equations allow now for the existence of an interesting minimum:

$$y'_c \simeq \frac{1}{\Lambda_f}\sqrt{\frac{g'_{UD}\delta\tilde{\mu}_D^2 - \tilde{\lambda}_D\delta\mu_U^2}{g_{UD}'^2 - 4\tilde{\lambda}_D\left(\lambda_U+\lambda_{UU}\right)}}, \quad y'_b \simeq \frac{1}{\Lambda_f}\sqrt{\frac{g'_{UD}\delta\mu_U^2 - 2\left(\lambda_U+\lambda_{UU}\right)\delta\tilde{\mu}_D^2}{g_{UD}'^2 - 4\tilde{\lambda}_D\left(\lambda_U+\lambda_{UU}\right)}}. \tag{3.16}$$

Remarkably, $y'_{c,b}$ take on $\mathcal{O}(1)$ values, and thus, the masses of the charm and bottom quarks can be given a dynamical origin within the model. This procedure could potentially be continued into higher orders of the expansion, targeting the different parameters in the potential, aiming for solutions like those in Eq.(3.16) for the remaining variables. It should be however noted, that there is a lot of freedom in the 13-dimensional parameter space, and even though general inferences can be made looking at the structure of the potential, like the dimensionful parameters necessarily requiring some degree of suppression, it cannot be reasonably expected to explore each and every possibility in this fashion. Multitude of divergent paths can be taken right from the first choice, given in the above example by Eq.(3.14). Nevertheless, many such possibilities have been explored in the making of this work. It is relatively easy to give mass to the two heaviest quarks through $\mathcal{O}(1)$ expressions alike to those shown in Eq.(3.16). Afterwards, this procedure generically tends to lead to one of the two following outcomes: either the right mass is generated for the strange quark, resulting in the absence of quark mixing and vanishing masses for the first generation; or non-vanishing θ_a allows for mixing between the third and second generations, at the price of massless up, down and strange quarks and undetermined and vanishing mixing structure among the first and the second and third families respectively. To summarise, the enforcing of mild fine-tunings among the parameters of the scalar potential (such as those shown in Eq.(3.14)), is enough to account for the dynamical origin of the bottom and charm (and even the strange) masses, but the remaining quarks stay massless, and there is no way to accommodate the full mixing pattern exhibited by the SM.

But before taking these conclusions any further, let us turn next to the numerical study of the scalar potential. The analytical analysis was made challenging by the sheer amount of terms in the potential, which thankfully, will no longer be an issue. What will, however, still be troublesome, is the large region of 13-dimensional parameter space that we should aim to explore before giving a definite answer. Seeing as we were unsuccessful accommodating the whole flavour structure of the SM, we are now interested in the question of whether is it possible at all, and if it is, to which degree of detriment in the naturalness displayed by the potential of the data driven model, when compared to the above instances of MFV or the very flavour sector of the SM.

The numerical approach allows to effortlessly reparametrise the flavon vevs to the best of our experimental knowledge, without having to worry about messy analytical expressions:

$$V_{CKM} \simeq \begin{pmatrix} 1-\theta_c^2\lambda^2/2 & \theta_c\lambda & \theta_a\theta_c A\lambda^3\,\|\theta_b\rho - i\theta_d\eta\| \\ -\theta_c\lambda & 1-\theta_c^2\lambda^2/2 & \theta_a A\lambda^2 \\ \theta_a\theta_c A\lambda^3\,\|1-\theta_b\rho - i\theta_d\eta\| & -\theta_a A\lambda^2 & 1 \end{pmatrix},$$

$$y_u = y'_u\lambda^{7.59}, \quad y_c = y'_c\lambda^{3.30}, \quad y_d = y'_d\lambda^{7.05}, \quad y_s = y'_s\lambda^{5.05}, \quad y_b = y'_b\lambda^{2.50}, \tag{3.17}$$

where the whole set of Wolfenstein parameters in Eq.(1.23) has been adopted. Notice this time we are avoiding any technical difficulty by introducing four variables – $\theta_{a,b,c,d}$, offering now a complete description of the four physical observables in the CKM matrix

As a result of the above parametrisation, the flavour structure of the SM is exactly reproduced not only when the variables $y'_{u,c,d,s,b}$ and $\theta_{a,b,c,d}$ take $\mathcal{O}(1)$ values, but in fact, very close to 1, which will make future results easier to interpret. Notice we have purposefully removed the complex phase from the entries of the CKM matrix, as the position of the minima only depends on their moduli (see Eq.(3.9)), and will be thus irrelevant for the present analysis.

To comb through the vast parameter space, a Monte Carlo based approach will be employed, randomly sampling different sets of the parameters appearing in the scalar potential. Each set will be judged on the proximity of the nearest minimum to the point

$$(y'_u, y'_c, y'_d, y'_s, y'_b, \theta_a, \theta_b, \theta_c, \theta_d) = (1, 1, 1, 1, 1, 1, 1, 1, 1)\,, \tag{3.18}$$

which, after the new parametrisation, harbours the SM flavour structure. The minimisation is carried by numerical means, with bias towards minima with larger second derivatives, indicative of better stability. As previously discussed, with the structure of the potential in Eq.(3.10), we are after the generation of bounded-from-below, mexican-hat like, one-dimensional cuts of the potential for each of the variables in play, whose minimum shall lie at the point in Eq.(3.18).

There is an important point which should be remarked, available computational resources allow for the exploration of the parameter space only up to a certain degree of precision. We will not be able to claim we have found every possible solution, nor the best one, instead, we will just be answering the question of whether a desirable solution can be achieved at all, and drawing general conclusions regarding the extent of the fine-tunings required. In this situation, the distribution from which the parameters are being randomly sampled becomes a relevant matter, e.g. a flat distribution between 1 and -1 would bias the sampling of the quartic parameters towards non fine-tuned values. To better explore the parameter space, other distributions such as exponentials have also been implemented in combination to the latter.

The results of the analysis are scarce, but once the Monte Carlo algorithm provides a set of parameters with a minimum roughly neighbouring the point in Eq.(3.18), it is possible to iteratively improve its proximity up to the desired degree of accuracy. This can be done by selectively randomizing a handful of parameters at a time, targeting the specific variables which have been minimised further from 1, which ultimately reproduces their value in the SM. The best result found in this fashion requires the enforcing of the following hierarchies among the parameters of the scalar potential

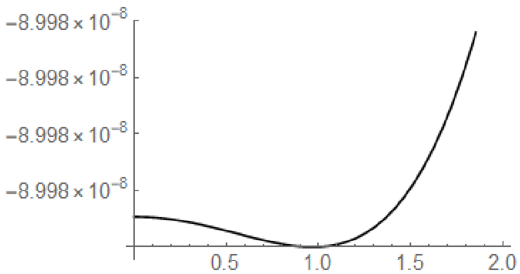
$$\begin{gathered}\tilde{\mu}_D = \delta\tilde{\mu}_D\lambda^2, \quad \mu_U = \delta\mu_U\lambda^3, \quad \mu_D = \delta\mu_D\lambda^4, \\ \lambda_U = \mathcal{O}(1), \quad \lambda_D = \mathcal{O}(1), \quad \tilde{\lambda}_D = \mathcal{O}(1), \\ g_{UD} = \delta g_{UD}\lambda^3, \quad g'_{UD} = -\delta g'_{UD}\lambda^6, \quad g'_{DD} = \delta g'_{DD}\lambda^5, \\ \lambda_{UU} = -\delta\lambda_{UU}\lambda^{10}, \quad \lambda_{DD} = \delta\lambda_{DD}\lambda^{11}, \quad \lambda_{UD} = \delta\lambda_{UD}\lambda^{12}, \quad \lambda'_{DD} = \delta\lambda'_{DD}\lambda^{11},\end{gathered} \tag{3.19}$$

where the suppression has been factorised in terms of λ so that $\mathcal{O}(1)$ values correspond to the fractions $\delta\mu/\Lambda_f$ and the quartic parameters $\delta\lambda$ and δg. Remarkably, the scalar potential constructed out of the above parameters is minimised by the nine-dimensional point in Eq.(3.18).

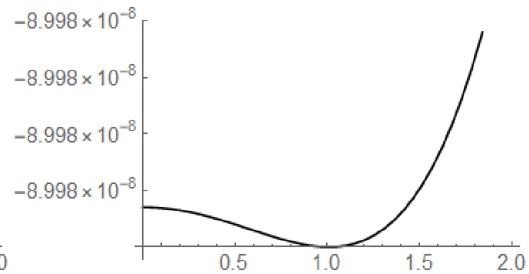
For the sake of illustrating this statement, one dimensional cuts of the potential in which all but one variable have been fixed to their values at the minimum – i.e. 1, are shown for the Yukawa eigenvalues and the mixing observables in **Figures 3.2** and **3.1** respectively. The good mexican-hat like behaviour of the potential is showcased in both, being always minimised when the corresponding variable takes very-close-to-1 values.

We conclude that it is indeed possible to dynamically generate the full flavour structure of the hadronic sector, including quark masses and mixings, within the data driven flavour model, albeit at the price of the fine-tunings introduced in Eq.(3.19). Upon comparison with those necessary to accurately describe the quark sector within the bare SM (see Eq.(3.17)), we cannot claim the data driven model provides a more natural explanation to the origin of flavour that the one already present in the latter. In fact, the situation has worsened to some extent, with a larger quantity of parameters displaying similar or greater degrees of fine-tuning.

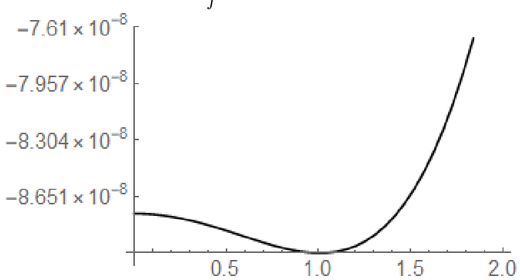
Nevertheless, the data driven flavour model succeeds in the comparison against the MFV scenarios previously reviewed in this work. When bi-fundamental flavons where considered within the MFV framework, the analysis led to vanishing or undetermined mixing angles and a single massive quark in the up and down sectors at the renormalisable level. The situation improved ever so sightly after the consideration of non-renormalisable operators, allowing masses for the lighter families. Nonetheless, the correct pattern of masses and mixings could not be possibly accounted for. On the other hand, when fundamental flavons were considered at the renormalisable level, a strong mass hierarchy arose, singling two massive quarks in each sector, together with a non-vanishing mixing angle that could be identified with



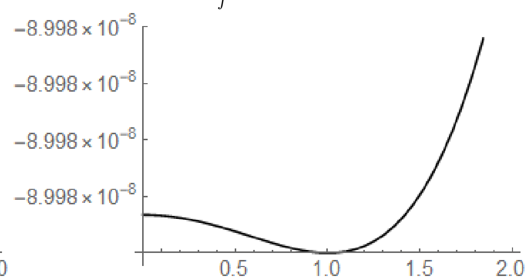
(a) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(y'_u)$



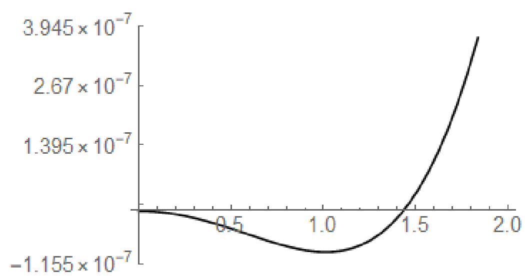
(b) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(y'_d)$



(c) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(y'_c)$



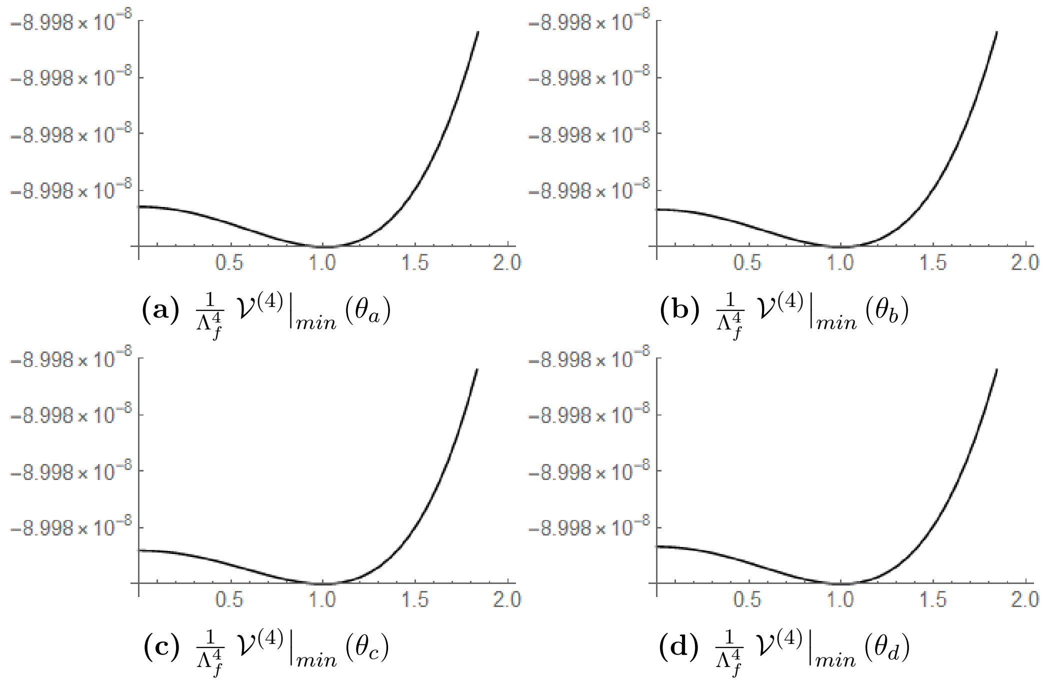
(d) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(y'_s)$



(e) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(y'_b)$

Figure 3.1: One dimensional plotting of the (rescaled) flavon scalar potential – $\frac{1}{\Lambda_f^4}\mathcal{V}^{(4)}$ – at its minimum. The observed quark masses in the SM are correctly reproduced when these variables minimise the potential near 1 values.

the rotation between the second and third families. However, not even non-renormalisable operators could provide with masses for the lighter quarks or fully account for the mixing pattern of the SM. In short, both of these scenarios cannot possibly account for the granularity of the flavour sector when



(a) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(\theta_a)$ **(b)** $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(\theta_b)$

(c) $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(\theta_c)$ **(d)** $\frac{1}{\Lambda_f^4}\,\mathcal{V}^{(4)}\big|_{min}\,(\theta_d)$

Figure 3.2: One dimensional plotting of the (rescaled) flavon scalar potential – $\frac{1}{\Lambda_f^4}\mathcal{V}^{(4)}$ – at its minimum. The fully fledged CKM mixing matrix is accurately reproduced when these variables minimise the potential near 1 values.

sourcing the Yukawa couplings. This is in stark contrast to the solution provided by the data driven flavour model, which, already at the renormalisable level, can provide a dynamical origin for every physical observable characterising flavour in the SM, as long as suitable fine-tunings are enforced.

A possible avenue for improvement potentially worth exploring in the future, could be the adoption of different sets of flavons (compatible with the flavour symmetry of the data driven model) other than the minimal resulting from the assignment of a single flavon per Yukawa coupling. In particular, fundamental flavons have shown their usefulness in the MFV framework when inducing hierarchies and non-trivial mixing sourced by the misalignment between those sharing the same vector space. Either a complete set of fundamental or a mixture of fundamental and bi-fundamental flavons could provide a more natural origin to the structure of the flavour sector. The analysis on the resulting phenomenology of such scenarios would speak for their viability as possible alternatives.

4

Conclusions

The main idea behind this work is the use of symmetry in the quest for an explanation to the family structure exhibited by the quark sector within the SM. The flavour symmetry here invoked, is the defining characteristic of the scenarios under discussion.

In a rather model-independent way, the assumption that at low energies, the Yukawa couplings constitute the only source of flavour and CP violation in beyond the SM theories, ensues the well agreement of the latter with experimental flavour data. This is the basis of the MFV hypothesis, implemented through effective Lagrangian techniques.

Taking the next natural step, we assume the flavour symmetry to be exactly realised at some energy scale Λ_f. As a consequence, the Yukawa couplings can only arise in this scenario as the vevs of new dynamical fields, the flavons, which have non-trivial transformations properties under the flavour group. This is because the insertion of the latter is needed so as to make the now $d > 4$ Yukawa operators invariant under the flavour symmetry. In this way, the structure of the Yukawa couplings is generated after the flavons develop a vev, spontaneously breaking the flavour symmetry. The relevant question then becomes what may be the scalar potential responsible for the non-zero vevs of the flavons, and even more so, may some of its minima naturally correspond to the SM structure of quark masses and mixing angles? In this work, we revisit the latter for MFV, ultimately addressing it for the Data Driven Flavour Model.

The simplest realisation is to consider each Yukawa coupling is sourced by a single scalar flavon field transforming in the bi-fundamental representation of the flavour group. From the point of view of effective Lagrangians, this possibility may correspond to the lowest order term in the inverse power expansion on the flavour scale $1/\Lambda_f$, involving a $d = 5$ effective Yukawa operator comprised by one flavon field and the usual SM field combination. The construction and analysis of the general flavour invariant scalar potential for these bi-fundamental flavons has been reviewed for both the two-family and the three-family case. For the latter and at the classical and renormalisable level, only vanishing or undetermined mixing angles are permitted. Certain fine-tuned regions of the potential parameter space allow for solutions with vanishing Yukawa couplings for all quarks except those in the heaviest family, which otherwise tend to be generically degenerate, but even then, mixing would still be absent. The addition of non-renormalisable higher order operators to the potential leads to the generation of masses for the lighter families, but ultimately fails at providing a realistic pattern of masses and mixings. These findings can be summarised by stating that the bi-fundamental flavons here considered cannot possibly account for the dynamical origin of the full flavour structure characterising the hadronic sector of the SM.

A second scenario has been reviewed in which each Yukawa coupling arises instead from the combined vevs of two scalar flavons transforming in the fundamental representation of the flavour group, as quarks do. In the effective Lagrangian language, a possible realisation could be that of the next-to leading order $d = 6$ Yukawa operators, now suppressed by two inverse powers of the flavour scale $1/\Lambda_f^2$. Again, the construction and analysis of the general scalar potential for these fundamental flavons has been reviewed for both, the two- and the three-family case. Due to the vector nature of the fundamental flavons and their structure within the resulting Yukawa operators, a strong mass hierarchy is unavoidable regardless of the consideration or not of non-renormalisable operators: only two quarks, one in the up and one in the down sector, get masses. In the three-family case, the natural choice is for the masses to be assigned to the top and bottom quarks of the third generation, being the heaviest. Non-vanishing mixing structure requires a departure from minimality, in that two distinct flavons need to be assigned to the up and down left-handed quarks, despite both belonging to the same representation of the flavour group (which means only one of them would be strictly needed to render the Yukawa interactions invariant). This is because mixing is sourced within this setup by the misalignment in flavour space of the flavons associated to the left-handed quark $SU(2)_L$ doublet. As a consequence, only one mixing angle can possibly be generated. To offer a complete picture, for fundamental flavons, it follows that a strong mass hierarchy is induced in the quark spectrum, characterised by a single massive quark in each sector; whereas only one non-vanishing mixing angle is generated, which can be realistically identified with the rotation in the 23 sector of the CKM matrix. In the end, we are still missing the generation of masses for the lighter quarks and a complete description of the full mixing pattern, and in this sense, we must conclude again that this realisation cannot possibly account for the granularity of the quark flavour structure in the SM.

If something can be extracted from the above scenarios is that a departure from the most straightforward approaches seems to be warranted if we wish to dynamically accommodate the full quark flavour spectrum. The data driven flavour model is a bottom-up approach, built on the premise of strictly following the available experimental data regarding the flavour sector. Avoiding any fine-tuning on the Yukawa couplings, the quark mass hierarchy suggests that only the top Yukawa term should arise at the renormalisable level, as it involves the only $\mathcal{O}(1)$ parameter satisfying naturalness criteria in the flavour sector. The idea is, as it was before, to identify the largest flavour symmetry group arising in the limit of vanishing Yukawa couplings, only that now the top coupling is excluded from the limit. This symmetry is realised at some energy scale Λ_f and as a consequence of its new definition, the top Yukawa interaction no longer requires the insertion of any flavons, which are however still needed for the rest of the quarks. This is already a step in the right direction, the model distinguishes the third family from the two lighter ones from the very beginning, naturally describing a top Yukawa of order 1. A one-to-one correspondence is established between the vevs of the flavons and each of the Yukawa couplings. In this sense, the model has been kept as minimal as possible.

The construction of the most general flavour invariant scalar potential for the data driven model has been carried in this work. Unfortunately, its complexity and intricate dependences on the physical flavour observables make impractical any analytical attempt towards its analysis. Nonetheless, a simple lesson can be extracted from them: dynamically generating masses and mixings in the quark sector necessarily requires of some degree of fine-tuning among the parameters in the scalar potential. A numerical analysis was carried next, adopting a Monte Carlo based approach to randomly sample the parameters in the potential. Drawing from several distributions was required in order to ensure the relatively large 13-dimensional parameter space was evenly explored. Each iteration was judged on the proximity of the nearest minimum to the experimentally observed values for the physical observables in the flavour sector. In the end, the results of the analysis remarkably lead to a solution reproducing the full pattern of quark masses and mixings.

The fine-tunings required happen to be ultimately stronger than those compulsory for the description of quark flavour in the bare SM. Nevertheless, when compared to the above realisations of MFV, the data driven model leads to a substantial degree of improvement. While the first offer at best, a single mass splitting, separating the third from the first two families; and a single non-vanishing mixing angle; even after considering the addition of non-renormalisable terms to the potential, the data driven flavour model can provide a dynamical explanation for the full set of physical flavour observables, including the CP violating phase, already at the renormalisable level.

Despite the somewhat bittersweet ending, to what started as a quest to give a natural origin to the flavour puzzle, one cannot help but think about the exciting possibilities which remain to be explored within this field. The data driven flavour model here presented and analysed, has been shown to provide a significant improvement over generic instances of MFV, while still maintaining its status as a potentially testable theory in the $\sim TeV$ range. The departure from the minimal set of flavons here considered, could potentially lead to even further improvements in naturalness, constituting an alluring possibility to be explored in future works.

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